Solving a quadratic equation

a case study

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A pesky problem

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If that doesn't convince the admin type, what will?

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3. $x^2 - 3x - 1 = 0$ (sort of fancy... just right!)

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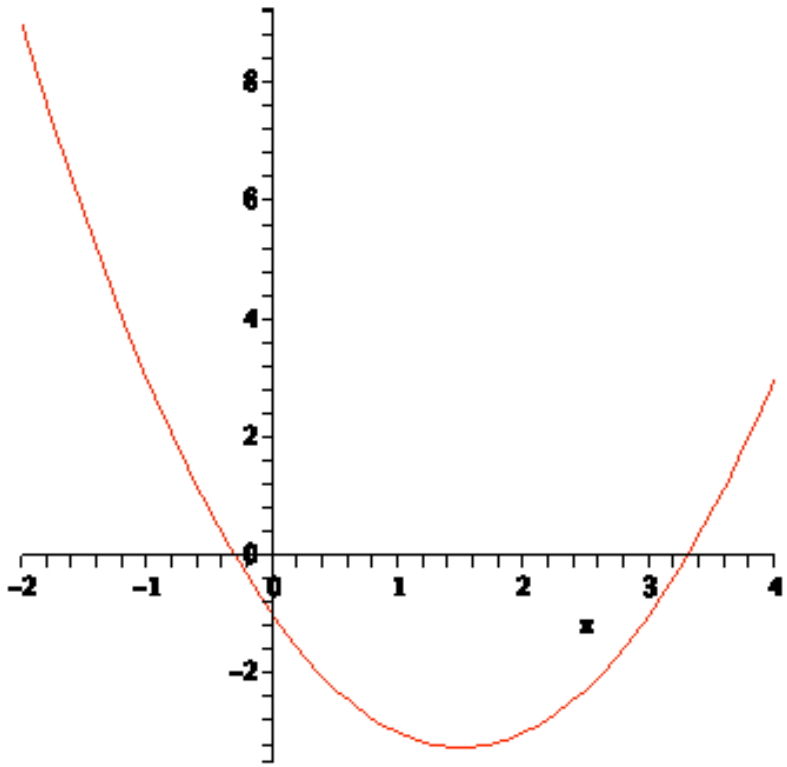

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"You want money for your one lousy graph?"

"Give the solution to 10 decimals, and we'll show you the money!"

"Oh, for @#%& sake!"

factor, factor, complete...

$$0 = x^2 - 3x - 1$$

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$$\begin{aligned} 0 &= x^2 - 3x - 1 \\ 0 &= x^2 - 3x + (-3/2)^2 - (3/2)^2 - 1 \end{aligned}$$

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Now let's solve it:

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Think this is enough to get the money?

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Not likely...

Pay Up!

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$x = 3/2 + \sqrt{13/4}$, or

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and $x = 3/2 - \sqrt{13/4}$, or

$x = -0.302775637731994646559610633735247973125648286922623106355$

Mathematical Proof

The final proof that we are Mathematicians?

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Give them the Magic Formula,

$$ax^2 + bx + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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and tell them to try this first next time...