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Dual homotopy invariants of G-foliations.
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This paper is an important contribution to the theory of characteristic invariants of foliations. In it, Sullivan's theory of minimal models is used to define dual homotopy invariants of foliations. These invariants are then related to the well-known secondary invariants of foliations and used to extend various results on independent variation of secondary invariants as well as to present a unified framework in which residue formulas for foliations with isolated singularities naturally arise.

The construction is the following. Let M be a smooth manifold which is simply connected and has finite Betti numbers (the last two assumptions are not essential) and let $\mathcal{F}$ be a G-foliation of codimension q. Applying the theory of minimal models to the Chern-Weil homomorphism and using the Bott vanishing theorem yields a map $h^{\#}\colon \pi^*(I(G)_{q'}) \to \pi^*(M) \cong \mathrm{Hom}(\pi_*(M), k)$. Here $I(G)_{q'}$ denotes the ring of invariant polynomials on G, truncated in degrees greater than q', a certain integer related to q, and $k = \mathbf{Q}$, $\mathbf{R}$ or $\mathbf{C}$ as required. The map $h^{\#}$ is shown to depend only on the G-concordance class of $\mathcal{F}$ and to be functorial. Moreover, if the normal bundle of $\mathcal{F}$ has an H-reduction there is a commutative diagram:

$$\begin{array}{ccc} \pi^*(I(G)_{q'}) & \xrightarrow{h^{\#}} & \pi^*(M) \\ {\scriptstyle\zeta}\uparrow & & \uparrow{\scriptstyle\mathcal{H}^*} \\ H^*(A(G,H)_{q'}) & \xrightarrow{\Delta_*} & H*(M) \end{array}$$

where $\mathcal{H}^*$ is the dual of the Hurewicz homomorphism and Δ_* is the restriction of the characteristic homomorphism of Bott-Haefliger, Kamber-Tondeur and others to a certain subalgebra of $H^*(W(\mathfrak{g},H)_{q'})$ for $W(\mathfrak{g},H)_{q'}$ a truncated relative Weil algebra. The algebra $\pi^*(I(G)_{q'})$ and the map ζ are explicitly computed and when H is the trivial group the map ζ is shown to be injective.

The author presents many applications of the above construction. A formula for evaluating elements in the image of $h^{\#}$ on Whitehead products is given. This in turn is used to give the first example of a nontrivial rigid secondary invariant of a foliation and to give residue formulas for foliations with isolated singularities. The existence of many Whitehead products of the classifying space $B\Gamma_G^q$ for G-foliations is then used to extend the results of various authors on independent variation of secondary invariants. Finally, it is shown that $\pi_*(B\Gamma_G^q)$ contains an uncountable number of linearly independent, free Lie subalgebras.

Reviewed by *Thomas E. Duchamp*