Let $BR^q\Gamma$ be the Haefliger classifying space for Riemannian foliations, $BO(q)$ the classifying space for O(q)-bundles and $FR^q\Gamma$ the homotopy-theoretic fiber of the classifying map $\nu: BR^q\Gamma \to BO(q)$.

The author shows the $(q-1)$-connectivity of $FR^q\Gamma$, and, more generally in the classifying theory of $G$-foliated microbundles for $G$ a closed subgroup of $GL(q, \mathbb{R})$, of the corresponding $F\tilde{\Gamma}^q_G$. For $F\tilde{\Gamma}^q_{\text{Sl}_2}$, this is a result of Haefliger. For $F\tilde{\Gamma}^q_{4k+3}$, this is the best possible result, according to a vanishing theorem of Pasternack.

It has applications to the existence of nontrivial Whitehead products in the homotopy of $BR^q\Gamma$ and to the nonnullity of the image of some secondary characteristic classes in $H^*(FR^q\Gamma, \mathbb{R})$.

The method consists in an explicit construction of an integrable homotopy out of a homotopy of $G$-foliations with trivial $G$-structures in order to apply the Gromov-Phillips-Haefliger theorem.