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Hurder, Steven
Independent rigid secondary classes for holomorphic foliations.
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A foliation $\mathcal{F}$ on a smooth manifold M is said to be transversely holomorphic if $\mathcal{F}$ is locally defined by submersions into $\mathbf{C}^n$, and the associated transition functions are biholomorphic (i.e. have values in the topological groupoid $\Gamma_n^{\mathbf{C}}$ of germs of local analytic automorphisms of $\mathbf{C}^n$). The purpose of this paper is to study the characteristic classes of transversely holomorphic foliations.

First, the universal Chern classes for transversely holomorphic foliations of codimension n are shown to be independent up to degree $2n$; in degrees $> 2n$ they must vanish. Namely, the author shows the following proposition. Let $B\Gamma_n^{\mathbf{C}}$ be Haefliger's classifying space of transversely holomorphic foliations of codimension n, and let $\nu\colon B\Gamma_n^{\mathbf{C}} \to B\mathrm{U}(n)$ be defined by taking the differential.

Proposition: The map $\nu^*\colon H^m(B\mathrm{U}(n);\mathbf{Q}) \to H^m(B\Gamma_n^{\mathbf{C}};\mathbf{Q})$ is injective for all $m \le 2n$.

Using this result, the author then constructs for all even complex dimensions the first examples of transversely holomorphic foliations for which a set of rigid secondary classes is nontrivial in $H^*(F\Gamma_n^{\mathbf{C}})$, where $F\Gamma_n^{\mathbf{C}}$ is the homotopy fibre of the map $\nu\colon B\Gamma_n^{\mathbf{C}} \to B\mathrm{U}(n)$. He gives such examples using his dual homotopy techniques [the author, Topology **20** (1981), no. 4, 365–387: MR 83a:57039] and a detailed study of the topology of the map $\nu\colon B\Gamma_n^{\mathbf{C}} \to B\mathrm{U}(n)$.

Applications of the independence results stated above are given to the study of $B\Gamma_2^{\mathbf{C}}$ and to the study of the space of foliations on an open manifold.

Reviewed by *Masahisa Adachi*