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The classifying space of smooth foliations.

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Thurston constructed a family of codimension one foliations on S^3 for which the Godbillon-Vey class takes on a continuous range of values. The author gives far-reaching generalizations of this result. He shows that on certain spheres there exists an infinite set of codimension q foliations on S^n which are distinct up to homotopy and concordance, but whose tangential subbundles are all homotopic as embedded subbundles. In the paper this is proved for certain $n >> q$ and q an even number ≥ 10. The proof is based on the existence of surjective homomorphisms of $\pi_n(B\Gamma_q)$ onto large Euclidean spaces, where $B\Gamma_q$ denotes the classifying space of smooth codimension q foliations. The author's idea, here as in his earlier important contributions to the subject, is as follows. A foliated manifold M with a nontrivial secondary class is classified by a map $f\colon M \to B\Gamma_q$ with nontrivial image $f_* \subset H_*(B\Gamma_q;\mathbf{Z})$. In many cases f can then be modified by CW-space techniques to produce nontrivial spherical cycles in $H_*(B\Gamma_q;\mathbf{Z})$. The corresponding homotopy classes in $\pi_*(B\Gamma_q)$ generate then a free graded Lie subalgebra which makes those groups so enormous. The method also yields further independence results for the secondary classes, in addition to those previously established by Baker, Heitsch, Kamber and the reviewer.

Reviewed by *Ph. Tondeur*