
MR873431 (88m:46073)
Hurder, Steven (1-ILCC); **Olesen, Dorte** (DK-CPNH); **Raeburn, Iain** (5-NSW); **Rosenberg, Jonathan** (1-MD)
The Connes spectrum for actions of abelian groups on continuous-trace algebras.
Ergodic Theory Dynam. Systems **6** (1986), no. 4, 541–560.
46L55 (22D25 22D40 54H20 58F11)

Let (A, G, α) be a C^*-dynamical system and $A \times_\alpha G$ the corresponding crossed product C^*-algebra. O. Bratteli [Duke Math. J. **46** (1979), no. 1, 1–23; MR 82a:46063] conjectured that if G is abelian, A is of type I, and the action is G-simple (i.e., A has no nontrivial G-invariant ideals) and has full Connes spectrum, then $A \times_\alpha G$ is simple. In the paper under review, the authors prove that there are numerous counterexamples, derived from topological dynamics, to Bratteli's conjecture, and that the dual topology of a crossed product algebra can be surprisingly complicated.

For an action as above, it follows from results of the reviewer [in *Operator algebras and their connections with topology and ergodic theory* (Buşteni, 1983), 152–169, Lecture Notes in Math., 1132, Springer, Berlin, 1985; MR 86j:46066] that there is one common isotropy subgroup H for the action of G on the dual of A, and one subgroup S of H which is most relevant for the dual topology of $A \times_\alpha G$. The authors first prove (Proposition 1.3) that if, in addition, A is of continuous trace, then the primitive ideal space of $A \times_\alpha G$ is homeomorphic to the quasi-orbit space of the dual of $A \times_\alpha S$, under the action of G.

Furthermore, utilizing "realization" theorems as in the article of J. Phillips and Raeburn [J. Operator Theory **11** (1984), no. 2, 215–241; MR 86m:46058], they prove (Proposition 3.2) that if G is a direct product $S \times K$, and E is a principal $\hat{S}$-bundle over X, with an action of K commuting with that of $\hat{S}$, then there is an action of G on $A = C_0(X, \mathcal{K})$ (where $\mathcal{K}$ denotes the compact operators on a separable Hilbert space) such that the dual of $A \times_\alpha S$ and E are homeomorphic via a G-equivariant homeomorphism. Thus, if the action of G on X is minimal while the action of G on E has one dense orbit but is not minimal, a counterexample to Bratteli's conjecture will follow.

In Section 4, the authors exhibit two classes of dynamical systems with the requisite properties:

the first involves horocycle flows; for the second, they prove (Theorem 4.8) that if X is a compact metric space with a free minimal $\mathbf{R}$-action which is not uniquely ergodic, then there exists a continuous 1-cocycle for the flow such that the corresponding skew-product flow on $X \times \mathbf{R}$ has at least one dense orbit, but is not minimal.

The paper contains many other results, in particular information concerning the various spectra for the action of an abelian group.

Reviewed by *Elliot C. Gootman*