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Ergodic theory and Weil measures for foliations.

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Moussu-Pelletier and D. Sullivan asked whether a codimension-one, C^2 foliation with no leaves of exponential growth must have zero Godbillon-Vey class. Duminy (unpublished) proved the stronger, more natural result that a codimension-one, C^2 foliation without resilient leaves of a compact manifold must have zero Godbillon-Vey class. This paper begins with a study of the ergodic theory of foliations and especially the properties of measurable cocycles over discrete metric equivalence relations obtained from foliations. Using these ergodic theory techniques and J. Heitsch and Hurder's methods of Weil measures [J. Differential Geom. **4** (1984), no. 2, 291–309; MR 86i:57032] the authors prove that for a codimension-n, C^2 foliation on a smooth manifold M, if the measurable equivalence relation defined by the leaves of $\mathcal{F}$ is amenable, then all the residual secondary classes in degrees greater than $2n+1$ must vanish. Since a foliation for which the set of leaves with positive exponential growth type is a set of measure zero, is amenable, it follows that all the residual secondary classes in degrees greater than $2n+1$ in such foliations must vanish. Further by results of Hurder [ibid. **23** (1986), no. 3, 347–365; MR 87m:57030], if almost every leaf has subexponential growth, the secondary classes in degree $2n+1$ vanish. Thus as a corollary the authors obtain an affirmative solution to the Moussu-Pelletier, Sullivan question for arbitrary codimension: in a codimension-n, C^2 foliation of a smooth manifold M for which almost every leaf has subexponential growth, all secondary classes of the foliation vanish. The Roussarie examples [see C. Godbillon and V. Vey, C. R. Acad. Sci. Paris Sér. A-B **273** (1971), A92–A95; MR **44** #1046] and W. P. Thurston's examples [Bull. Amer. Math. Soc. **78** (1972), 511–514; MR **45** #7741] give examples of codimension-n, amenable foliations $(n=1)$ with a nonvanishing class of degree $2n+1$. In these examples the leaves have exponential growth.

The authors' results can be reformulated in terms of the von Neumann algebra $\mathcal{M}(\mathcal{F})$ associated to the foliation $\mathcal{F}$. If $\mathcal{F}$ is a codimension-n, C^2 foliation whose von Neumann algebra is approximately finite, then all residual secondary classes for $\mathcal{F}$ in degrees greater than $2n+1$ must vanish. If $\mathcal{F}$ is a C^2 foliation of a smooth manifold M without boundary and $\mathcal{M}(\mathcal{F})$ is of type I then all residual secondary classes for $\mathcal{F}$ vanish.

Reviewed by *John Cantwell*