

Item: 1 of 1 | [Return to headlines](#)[MSN-Support](#) | [Help Index](#)Select alternative format: [BibTeX](#) | [ASCII](#)**MR1047276 (91c:58134)****[Hurder, Steven](#) (1-ILCC)****Eta invariants and the odd index theorem for coverings.***Geometric and topological invariants of elliptic operators* (Brunswick, ME, 1988), 47–82, *Contemp. Math.*, 105, Amer. Math. Soc., Providence, RI, 1990.[58G12](#) ([19K56](#) [46L80](#) [57R20](#))

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The author discusses odd index theorems on noncompact manifolds, analogous to the results of Atiyah and Singer on covering spaces, of Connes on measured foliations, and of the reviewer on regularly exhaustible manifolds. Let $\tilde{M}$ be a Galois covering with group Γ of a compact manifold M . Let $\tilde{D}$ be a Dirac-type operator on $\tilde{M}$ and let $\varphi: \tilde{M} \rightarrow U(n)$ be a unitary-valued multiplier on $\tilde{M}$. The aim of the paper is to analyse the index theory of the Toeplitz operators $T_\varphi = PM_\varphi P$ acting on L^2 , where P is the positive spectral projection of $\tilde{D}$ and M_φ is multiplication by φ . As is now becoming familiar, there is a problem related to the continuous spectrum of $\tilde{D}$ near zero, which forces one to consider also the “smoothed Toeplitz operators” obtained by replacing P by a smooth function of D approximating it.

If the multiplier φ is Γ -periodic, this analysis is exactly parallel to the Atiyah index theorem for covering spaces; alternatively, if Γ is amenable and φ is arbitrary, then the analysis is parallel to that involved in the reviewer’s work mentioned above. A major contribution of the paper is a construction which “interpolates” between these two extremes. The author defines a multiplier φ to be Γ -almost periodic if the closure of the Γ -translates of φ is compact (in a suitable topology) and the action of Γ on this closure is uniquely ergodic. An argument using the uniqueness of Haar measure shows that multipliers φ arising from flat vector bundles on M are Γ -almost periodic. The author proves a Toeplitz index theorem for Γ -almost periodic multipliers (and certain related but larger classes). Moreover, he remarks that the hypothesis of unique ergodicity allows one to control the way the smoothed Toeplitz operators approximate to the unsmoothed; this was also observed in a slightly different context by P. S. Muhly, I. F. Putnam and J. Xia [*Bull. Amer. Math. Soc.* **21** (1989), no. 1, 47–50; [MR 90a:47070](#)].

The author applies his index theorem to the task of proving that the relative η -invariant is equal

to a Γ -Toeplitz index (cf. the paper by R. G. Douglas, the author and J. Kaminker [ibid. **21** (1989), no. 1, 83–88; [MR 90g:58128](#)]). The set-up for this result is as follows: a Dirac operator D on M is coupled to a trivialized flat bundle V . There is then a path of coupled Dirac operators running from D_0 (where the coupling is defined by the trivialization) to D_1 (where the coupling is defined by the flat structure), and the relative η -invariant is by definition the integral of the continuous component of the derivative $(d/dt)\eta(D_t)$ [M. F. Atiyah, V. K. Patodi and I. M. Singer, Math. Proc. Cambridge Philos. Soc. **79** (1976), no. 1, 71–99; MR **53** #1655c]. Now the difference between the trivial and the flat structures on V is measured by a Γ -almost periodic multiplier φ on $\tilde{M}$, and the author proves that the relative eta invariant is equal to the Γ -index of the Toeplitz operator T_φ , by considering the variation with t of a “Type II” η -invariant defined directly on $\tilde{M}$.

The paper contains a number of misprints and minor slips which suggest that it was written rather hastily; but these should be obvious to the careful reader.

{For the entire collection see [90k:58206](#)}

Reviewed by [John Roe](#)

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