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Deformation rigidity for subgroups of $\mathrm{SL}(n, \mathbf{Z})$ acting on the n-torus.
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Let Γ be a subgroup of finite index in $\mathrm{SL}(n,\mathbf{Z})$, $G = \mathrm{Diff}^r(\mathbf{T}^n)$, and $R(\Gamma, G)$ the space of homomorphisms of Γ to G. $R(\Gamma, G)$ has a natural topology as a closed subspace of a Fréchet manifold. Let $\varphi \in R(\Gamma, G)$ be the action of Γ on the n-torus coming from the linear action of Γ on $\mathbf{R}^n$. A C^k-deformation of φ is defined to be a C^k path $t \mapsto \varphi_t \in R(\Gamma, G)$, $0 \le t \le 1$. A C^k-deformation φ_t is said to be trivial [resp., locally trivial] if there is a C^k path $t \mapsto H_t \in G$ such that $H_t^{-1} \circ \varphi_t \circ H_t = \varphi$ for $0 \le t \le 1$ [resp., for $0 \le t < \varepsilon$ for some $\varepsilon > 0$]. The author announces and sketches a proof that for $n \ge 3$ and $r = \infty$ or $r = \omega$, every C^1-deformation of φ is trivial and every C^0-deformation is locally trivial. Counterexamples to this theorem are known for $n = 2$.

The proof of the theorem proceeds in two steps: first a path of homeomorphisms H_t is constructed which satisfies the above condition; then it is shown that H_t has the desired regularity. The first step is accomplished by observing that there is a dense set of points in $\mathbf{T}^n$ each of which is fixed by a finite index subgroup of Γ. By a theorem of G. A. Margulis [*Discrete subgroups of Lie groups*, Springer, to appear], the first cohomology of the isotropy group of such a point with coefficients in the linear isotropy representation must vanish. Then by a theorem of D. Stowe [Proc. Amer. Math. Soc. **79** (1980), no. 1, 139–146; MR 81b:57035], the fixed points of the isotropy groups persist under deformation. The group Γ contains an element γ_0 which is hyperbolic and therefore $\varphi(\gamma_0)$ is Anosov. Now structural stability yields a path H_t, $0 \le t < \varepsilon$, of homeomorphisms conjugating $\varphi_t(\gamma_0)$ to $\varphi(\gamma_0)$. It is shown that H_t conjugates φ_t to φ on the dense set of points in $\mathbf{T}^n$ described above and therefore on all of $\mathbf{T}^n$. The proof of the regularity of H_t is more technical, and appeals to recent advances in the theory of smooth Anosov systems along with further cohomological properties of Γ.

The author observes that the triviality of C^k-perturbations (as opposed to deformations) is still an open question. The one place in the author's proof where this distinction is essential is in the application of Stowe's theorem.

Reviewed by *Garrett Stuck*