
MR1132311 (92k:58266)
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The longitudinal cocycle and the index of Toeplitz operators.
J. Funct. Anal. **101** (1991), *no. 1*, 120–144.
58G12 (19D55 19K56 46L80 47B35)

In this paper the authors study the index theory of Toeplitz operators along the leaves of a measured foliation. Such Toeplitz operators arise as follows: take a leafwise Dirac-type operator D, take its (leafwise) positive spectral projection P, and consider the operator $T_\varphi = PM_\varphi P$, where M_φ denotes multiplication by the smooth function φ on the manifold. If φ is invertible, T_φ is a Breuer-Fredholm operator in the von Neumann algebra of the foliation. What is its Breuer index?

A difficulty arises in applying the Connes-Skandalis longitudinal index theorem for foliations to solve this problem, namely that the commutator of two leafwise Toeplitz operators need not belong to the Connes C^*-algebra of the foliation (in contrast to the classical case, where the commutator of two Toeplitz operators must be compact). This phenomenon occurs because the leafwise Dirac operator may have continuous spectrum near zero. The authors deal with it by constructing a "smoothed" version of the Toeplitz extension and relating it to the original version; the commutator of two "smoothed" Toeplitz operators does belong to the Connes algebra.

After proving their index theorem the authors reformulate it in terms of cyclic cohomology. They conclude with some remarks on its relation to secondary invariants for elliptic operators; more details of this can be found in another paper of theirs [Invent. Math. **103** (1991), no. 1, 101–179; MR 91m:58152].

Reviewed by *John Roe*