
MR1236179 (94g:58165)
Hurder, Steven (1-ILCC)

Affine Anosov actions.

Michigan Math. J. **40** (1993), no. 3, 561–575.
58F15 (22E40 58F11)

In this interesting and well-written paper the author deals with affine Anosov actions of "large" discrete groups on tori and compact nilmanifolds. Recall that a compact nilmanifold is a quotient $X = N/C$ of a simply connected nilpotent Lie group N by a lattice $C \subset N$. A diffeomorphism $f\colon N/C \to N/C$ is said to be affine if there exist elements $\sigma \in \mathrm{Aut}(N)$ and $g \in N$ such that $\sigma(C) = C$ and $f = L_g \circ \sigma^*$, where σ^* is the automorphism of N/C induced by σ and L_g is the left translation of N/C by g. The automorphism σ^* is called the linear part of f. A smooth action φ of a group Γ on a manifold X is said to be Anosov if $\varphi(\gamma)$ is an Anosov diffeomorphism of X for some $\gamma \in \Gamma$.

Here the author studies the structure of the set of fixed points and the set $\Lambda(\varphi)$ of periodic points (= points with finite Γ-orbit) for affine Anosov actions. Any Anosov action of $\Gamma = \mathbf{Z}$ on a flat torus has at least one fixed point. However, the author proves that given any Anosov representation $\varphi_0\colon \mathbf{Z}^m \to \mathrm{SL}(n, \mathbf{Z}), m \geq 2$, there exists an Anosov action φ of some subgroup $\Gamma' \subset \mathbf{Z}^m$ of finite index with linear part φ_0 and no fixed points.

Now let Γ be a higher-rank lattice, viz. a lattice in a connected semisimple Lie group G, where the $\mathbf{R}$-rank of each factor of G is at least 2 and G has finite center. Then by a well-known theorem due to Margulis, $H^1(\Gamma, \rho) = 0$ for any finite-dimensional representation ρ of Γ. By making use of this result the author proves that any affine action φ of Γ on a compact nilmanifold X has a dense set $\Lambda(\varphi)$. Meanwhile, for each $n \geq 2$, there exists an Anosov action of some subgroup $\Gamma \subset \mathrm{SL}(n, \mathbf{Z})$ of finite index on $\mathbf{T}^n$ with no fixed points.

Reviewed by *Alexander Starkov*