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Affine Anosov actions.
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In this interesting and well-written paper the author deals with affine Anosov actions of “large” discrete groups on tori and compact nilmanifolds. Recall that a compact nilmanifold is a quotient $X = N/C$ of a simply connected nilpotent Lie group $N$ by a lattice $C \subset N$. A diffeomorphism $f: N/C \to N/C$ is said to be affine if there exist elements $\sigma \in \text{Aut}(N)$ and $g \in N$ such that $\sigma(C) = C$ and $f = L_g \circ \sigma^*$, where $\sigma^*$ is the automorphism of $N/C$ induced by $\sigma$ and $L_g$ is the left translation of $N/C$ by $g$. The automorphism $\sigma^*$ is called the linear part of $f$. A smooth action $\varphi$ of a group $\Gamma$ on a manifold $X$ is said to be Anosov if $\varphi(\gamma)$ is an Anosov diffeomorphism of $X$ for some $\gamma \in \Gamma$.

Here the author studies the structure of the set of fixed points and the set $\Lambda(\varphi)$ of periodic points ($=$ points with finite $\Gamma$-orbit) for affine Anosov actions. Any Anosov action of $\Gamma = \mathbb{Z}$ on a flat torus has at least one fixed point. However, the author proves that given any Anosov representation $\varphi_0: \mathbb{Z}^m \to \text{SL}(n, \mathbb{Z}), m \geq 2$, there exists an Anosov action $\varphi$ of some subgroup $\Gamma ' \subset \mathbb{Z}^m$ of finite index with linear part $\varphi_0$ and no fixed points.

Now let $\Gamma'$ be a higher-rank lattice, viz. a lattice in a connected semisimple Lie group $G$, where the $\mathbb{R}$-rank of each factor of $G$ is at least 2 and $G$ has finite center. Then by a well-known theorem due to Margulis, $H^1(\Gamma, \rho) = 0$ for any finite-dimensional representation $\rho$ of $\Gamma$. By making use of this result the author proves that any affine action $\varphi$ of $\Gamma$ on a compact nilmanifold $X$ has a dense set $\Lambda(\varphi)$. Meanwhile, for each $n \geq 2$, there exists an Anosov action of some subgroup $\Gamma \subset \text{SL}(n, \mathbb{Z})$ of finite index on $\mathbb{T}^n$ with no fixed points.

Reviewed by Alexander Starkov

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