A small (in norm) perturbation of an elliptic operator cannot change its spectrum very much; if the perturbation is topologically nontrivial it must change the index. This tension has been exploited in several contexts, beginning with the work of Gromov and Lawson on the positive scalar curvature problem. The article under review makes use of the same principle.

The author considers selfadjoint elliptic operators (of geometric type) on a cover or along the leaves of a foliation of a compact manifold. The foliation is supposed to be equipped with an invariant transverse measure, defining a trace on its $C^*$-algebra and allowing one to measure “renormalized” spectral densities. By considering the odd version of Connes’ measured foliation index theorem, the author obtains estimates for the renormalized spectral density function of a geometric operator.

The estimates imply, for example, that if the operator has nonzero index when paired with a $K^1$ class which is “almost flat” in an appropriate sense, then the spectrum is the entire real line. However, the work also has a quantitative aspect: the author introduces a “noncommutative isoperimetric function” which measures how difficult it is to represent the $K^1$-class by a matrix of curvature $< \varepsilon$; he then obtains lower bounds for the spectral density in terms of this function.

{For the entire collection see 94b:00038}