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Exotic index theory and the Novikov conjecture.

Novikov conjectures, index theorems and rigidity, *Vol.* 2 (*Oberwolfach*, 1993), 255–276, *London Math. Soc. Lecture Note Ser.*, 227, *Cambridge Univ. Press, Cambridge*, 1995.
58G12 (19K35 19K56 57R67)

This is an analytic approach to the Novikov conjecture using techniques from the coarse index theory of N. Higson and J. Roe [in *Novikov conjectures, index theorems and rigidity, Vol. 2 (Oberwolfach, 1993)*, 227–254, Cambridge Univ. Press, Cambridge, 1995; MR 97f:58127; MR 97f:58127] and ideas from coarse topology (Ferry, Weinberger, and others). The basic method is that of A. S. Mishchenko, in the KK-theory formulation of G. G. Kasparov. A novel feature is the use of fibered versions of coarse analytic constructions. Let M be a manifold with fundamental group Γ. M is called ultraspherical if there is a map of $\tilde{M}$ to $\mathbf{R}^n$ of nonzero degree and uniformly bounded gradient. Let $\tilde{M}\Gamma$ be the balanced product $(\tilde{M}\times\tilde{M})/\Gamma$, which is a bundle over M with projection π. M is Γ-ultraspherical if $\tilde{M}\Gamma$ admits a fiber-preserving map to the tangent bundle of M which has the above property on each fiber. The main result is Theorem 1.1. The body of the paper shows that its statement should be corrected as follows: Let Γ be a group whose classifying space is a complete Riemannian spin manifold M which is Γ-ultraspherical. Then the assembly map $\beta\colon K_*(M)\to K_*(C_r^*\Gamma)$ is rationally injective. If M is ultraspherical of degree ± 1 then β is injective. This allows verification of the Novikov conjecture for new classes of non-finitely presented groups.

The proof involves analysis of the family of Dirac operators on the fibers of $\tilde{M}\Gamma$. (Rational injectivity without the spin hypothesis may be obtained by substituting the signature operator.) This requires the introduction of a fibered Roe algebra $C^*(\tilde{M}\Gamma,\pi)$ and a fibered Higson corona $\partial_\pi\tilde{M}\Gamma$. A key construction is the coarsening map $K_q(C_r^*\Gamma)\to K_q(C^*(\tilde{M}\Gamma,\pi))$; its range is much more manageable than its domain. The image of this is paired with a "dual Dirac" element of $KK_{p+1}(C^*(\tilde{M}\Gamma,\pi),C(M))$ depending on $u\in K^p(\partial_\pi\tilde{M}\Gamma)$, giving maps $K_q(C_r^*(\Gamma))\to K^{p+q+1}(M)$. Composing with the assembly map and Poincaré duality yields the exotic index maps $K^*(M) \to K^{*+p+q+1}(M)$. The notion of a coarse Bott (or Thom) class is introduced. When M is compact this is a $\Theta \in K^*(\tilde{M}\Gamma)$ such that there exists $u_\Theta \in K^{*+1}(\partial_\pi \tilde{M}\Gamma)$ with $\Theta = \delta u_\Theta$, and the pairing of Θ with the Dirac operator on each fiber is ± 1. If M is Γ-ultraspherical of degree one then $\tilde{M}\Gamma$ admits a coarse Bott class. The main theorem follows by applying a families version of an index theorem of G. Yu ["K-theoretic indices of Dirac type operators on complete manifolds and the Roe algebra", Preprint, Math. Sci. Res. Inst., Berkeley, 1991; per bibl.] to show that the exotic index corresponding to u_Θ is injective. A sketch proof of an important theorem of G. Carlsson and E. K. Pedersen [Topology **34** (1995), no. 3, 731–758; MR 96f:19006] is given using these methods.

{For the entire collection see 96m:57003}

Reviewed by *John G. Miller*