

Item: 1 of 1 | [Return to headlines](#)[MSN-Support](#) | [Help Index](#)Select alternative format: [BibTeX](#) | [ASCII](#)**MR1878608 (2003a:57051)****[Heitsch, James L.](#) (1-ILCC); [Hurder, Steven](#) (1-ILCC)****Coarse cohomology for families. (English summary)***[Illinois J. Math.](#) 45* (2001), *no. 2*, 323–360.[57R30 \(53C23 55N30 57R32 58H10\)](#)[Journal](#)[Article](#)[Doc Delivery](#)[References: 29](#)[Reference Citations: 2](#)[Review Citations: 0](#)

Coarse cohomology for metric spaces was introduced by J. Roe using anti-Čech systems of coverings, where successive coverings are “coarser” rather than “finer”. In this paper, the authors generalize Roe’s construction to parametrized families of metric spaces satisfying certain conditions. The main example is given by foliations: let F be a foliation of a compact manifold M whose holonomy groupoid \mathcal{G}_F is Hausdorff. Endow the holonomy cover \tilde{L}_x of the leaf through $x \in M$ with the distance d_x induced from any choice of metric on M . Then $(\tilde{L}_x, d_x)_{x \in M}$ is a family of metric spaces with well-defined coarse cohomology.

In the case of foliations there are several other equivalent definitions: the coarse de Rham theory, defined using a bicomplex $A_{k,l}(F)$ of smooth k -forms in $\mathcal{G}_{l+1} = \{(y_1, \dots, y_{l+1}) \in \mathcal{G}_F^{l+1}: s(y_1) = s(y_j) \forall j\}$; the coarse Čech cohomology, defined using “good” covers of \mathcal{G}_F in the sense of Leray; the coarse Alexander-Spanier cohomology, defined using a bicomplex $C_c^{k,l}(F)$ of locally bounded functions $\varphi: \mathcal{G}_{l+1}^{k+1} \rightarrow \mathbb{R}$ with some assumptions on $\text{supp } (\varphi)$.

Coarse cohomology is computed in a number of examples of foliations.

Reviewed by [Jean-Louis Tu](#)

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