PROBLEM SESSION – FOLIATIONS 2012

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Contents

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1. Introduction

This is a collection of problems from the theory of foliations and related areas, proposed by the participants in the conference "Foliations 2012" held in Lodz, Poland during June 25–30, 2005.

"Foliation problem sets" have a long tradition: Stanford 1976, compiled by Mark Mostow and Paul Schweitzer [72]; Rio de Janeiro 1976, compiled by Paul Schweitzer [82]; Rio de Janeiro 1992, compiled by Rémi Langevin [47]; Santiago do Compostela 1994, compiled by Xosé Masa and Enriqué Macias-Virgós [56]. There was no general problem set published after the meeting Warsaw 2000, although the survey [39] formulated open problems in the area of foliation dynamics and secondary classes, and the unpublished problem set [40] was prepared for the Conference Geometry and Foliations" held in Kyoto, Japan in September 2003.

2. Jesús Álvarez López

2.1. **Furstenberg type structure theorem for distal foliated spaces.** A celebrated work of H. Furstenberg [23] shows that an ergodic distal action on a Borel probability space is isomorphic to an inverse limit of a tower of equicontinuous factor actions. The Furstenberg Structure Theory was extended to minimal Borel actions on compact spaces in the works of Lindenstrauss [55] and Akin, Auslander and Glasner [1]. The notion of a distal pseudogroup action was defined by Hurder in [43], [15, section 4] and [44, section 3], and by A. Biś and P. Walczak in [3]. The notion of a compactly generated pseudogroup was defined by defined by A. Haefliger in [33], as a generalization of the pseudogroups obtains from nice sections to foliations of compact manifolds.

PROBLEM 2.1. *Is there a (Borel) version of the Furstenberg Structure Theory for compactly generated pseudogroups whose action is distal?*

Perhaps the main point of this problem is the concept of completeness for pseudogroups, as proven by É. Salem [32, 81] and [69, Appendix D] for isometric actions. The reason is that, for distal actions, a key ingredient of Furstenberg's proof is played by the Ellis group (see [1]), which is defined by taking pointwise limits of the transformations. However, a sequence of local transformations of the pseudogroup may have smaller and smaller domains, obtaining a point domain at the limit. A pseudogroup version of completeness for pseudogroups would guarantee that the transformations of the pseudogroup can be locally extended to large enough open sets, so that an analogous version of the "Ellis pseudogroup" would exist, and then Furstenberg's arguments can be easily adapted. But distality does not seem to imply completeness, and therefore some additional argument is needed. Perhaps a weaker notion of completeness, which holds for all compactly generated distal pseudogroups, could be enough to extend Furstenberg's arguments.

3. Andrzej Biś

3.1. **Multifractal analysis of dynamics of foliated manifolds.** In the study of chaos and complexity of foliated manifolds, one often encounters invariant sets with very complicated geometry which reflect where the dynamics concentrate on. The multifractal analysis approach arises both in dimension theory and dynamical systems [73], and essentially concerns decomposition of a set of fractal nature into subsets with certain properties. This approach can be applied to foliation theory.

Geometric entropy was defined for a C^1-foliation $\mathcal{F}$ of a compact manifold M by Ghys-Langevin-Walczak in [26], and it has since become one of the principal numerical invariants for the dynamics of foliations (see Walczak [94] or Hurder [43, 44].) Let $\mathcal{T} \subset M$ be a complete transversal to $\mathcal{F}$ with

$\mathcal{G}_{\mathcal{F}}$ the induced compactly generated pseudogroup acting on $\mathcal{T}$, with a fixed generating set $\mathcal{G}_{\mathcal{F}}^{(1)}$. Given a subset $X \subset \mathcal{T}$, we say that $S = \{x_1, \ldots, x_\ell\} \subset X$ is (k, ϵ)-separated for $\mathcal{G}_{\mathcal{F}}$ and X if

$$\forall\ x_i \neq x_j\ ,\ \exists\ g \in \mathcal{G}|X \text{ such that } \|g\| \leq k\ \&\ d_{\mathcal{T}}(g(x_i), g(x_j)) \geq \epsilon$$

Here, $\|g\| \leq k$ means that g can be written as a composition of at most k elements of $\mathcal{G}_{\mathcal{F}}^{(1)}$. Then set

$$h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, X, k, \epsilon) = \max\ \#\{S \mid S \subset X \text{ is } (k, \epsilon)-\text{separated}\}$$

When $X = \mathcal{T}$, set $h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, k, \epsilon) = h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, \mathcal{T}, k, \epsilon)$.

Definition: [Ghys, Langevin and Walczak [26]] Let $\mathcal{G}_{\mathcal{F}}$ be a C^r-pseudogroup for $r \geq 1$, with generating set $\mathcal{G}_{\mathcal{F}}^{(1)}$. The *geometric entropy* of $\mathcal{G}_{\mathcal{F}}$ on $X \subset \mathcal{T}$ is

$$h(\mathcal{G}, \mathcal{G}_{\mathcal{F}}^{(1)}, X) = \lim_{\epsilon \to 0} \left\{ \limsup_{k \to \infty} \frac{\ln\{h(\mathcal{G}_{\mathcal{F}}, X, k, \epsilon)\}}{k} \right\}$$

The *geometric entropy* of $\mathcal{F}$ is defined to be $h(\mathcal{F}) \equiv h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, \mathcal{T}) < \infty$.

Brin and Katok introduced in [8, 28] a notion of local measure-theoretic entropy for maps. The concept of local entropy, as adapted to geometric entropy, was introduced in [43, Definition 13.3]. The set X in the above definition is not assumed to be saturated, so take $X = B(x, \delta) \subset \mathcal{T}$, the open δ–ball about $x \in \mathcal{T}$, to obtain a measure of the amount of “expansion” by the pseudogroup in an open neighborhood of x. Perform the same double limit process as used above for the sets $B(x, \delta)$, but then also let the radius of the balls tend to zero, to obtain:

Definition: The *local geometric entropy* of $\mathcal{G}_{\mathcal{F}}$ at x is

$$h_{loc}(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, x) = \lim_{\delta \to 0} \left\{ \lim_{\epsilon \to 0} \left\{ \limsup_{n \to \infty} \frac{\ln\{h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, B(x, \delta), k, \epsilon)\}}{k} \right\} \right\}$$

The local entropy determines the geometric entropy [43, Proposition 13.4]:

$$h_{loc}(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, x) \leq h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, \mathcal{T}) \quad , \quad h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, \mathcal{T}) = \sup_{x \in \mathcal{T}}\ h_{loc}(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, x)$$

In contrast with the extensively studied properties of local entropy for a single transformation (see [28, 84]), there are many open questions about the property of local geometric entropy (see [4]).

Definition: Assume that $h(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, \mathcal{T}) = A > 0$. For $\alpha \in [0, A]$, we say that a point $x \in X$ is a *point of $\alpha-$entropy* if $h_{loc}(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, x) = \alpha$. Define

$$K_\alpha = \{x \in \mathcal{T} \mid h_{loc}(\mathcal{G}_{\mathcal{F}}, x) = \alpha\}.$$

Then $K_\alpha \neq \emptyset$ for at least some values of α.

PROBLEM 3.1. *What is the topological structure of the sets K_α?*

PROBLEM 3.2. *When K_α is not empty? That is, find properties of the foliation $\mathcal{F}$ which imply there exists $x \in \mathcal{T}$ such that $h_{loc}(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)}, x) = \alpha$.*

PROBLEM 3.3. *Characterize the function $\alpha \mapsto$* Hausdorff Dimension(K_α).

Define an n-ball of radius r, centered at $x \in \mathcal{T}$, to be the set

$$B_n^{\mathcal{G}}(x, r) := \{y \in \mathcal{T} \mid d(h(x), h(y)) < r \ for\ all\ h \in \mathcal{G}_{\mathcal{F}}\ such\ that\ \|h\| \leq n\ and\ x, y \in Dom(h)\},$$

Definition: [4] For any $x \in X$ and a Borel probability measure μ on $\mathcal{T}$, the quantity

$$h_\mu^{\mathcal{G}}(x) = \lim_{\epsilon \to 0} \limsup_{n \to \infty} \left\{ -\frac{1}{n} \log \mu(B_n^{\mathcal{G}}(x, \epsilon)) \right\}$$

is called a **local upper μ-measure entropy** at the point x, with respect to $(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)})$, and

$$h_{\mu, \mathcal{G}}(x) = \lim_{\epsilon \to 0} \liminf_{n \to \infty} \left\{ -\frac{1}{n} \log \mu(B_n^{\mathcal{G}}(x, \epsilon)) \right\}$$

is called a **local lower μ-measure entropy** at the point x, with respect to $(\mathcal{G}_{\mathcal{F}}, \mathcal{G}_{\mathcal{F}}^{(1)})$.

For a Borel probability measure μ on $\mathcal{T}$, define

$$L_\alpha := \{x \in \mathcal{T} \mid h_\mu^{\mathcal{G}}(x) = \alpha\} \quad , \quad M_\alpha := \{x \in \mathcal{T} \mid h_{\mu,\mathcal{G}}(x) = \alpha\}.$$

which yields the decompositions (see [4]), $\mathcal{T} = \bigcup_\alpha M_\alpha = \bigcup_\alpha L_\alpha$.

PROBLEM 3.4. *What are properties of the families of sets $\{L_\alpha\}$ and $\{M_\alpha\}$?*

PROBLEM 3.5. *What is the relation between the sets $\{K_\alpha\}$, $\{L_\alpha\}$ and $\{M_\alpha\}$?*

4. Hélène Eynard-Bontemps

4.1. **Surface group representations.** Let $\mathcal{D}_+^r$ denote the set of orientation-preserving, C^r diffeomorphisms of the unit interval. The following result is shown in [20].

Theorem: For $r \geq 2$, any two representations of $\mathbb{Z}^2$ in $\mathcal{D}_+^r$ can be connected by a continuous path of representations of $\mathbb{Z}^2$ in $\mathcal{D}_+^1$. In other words, every two pairs of commuting C^r diffeomorphisms on the interval can be connected by a continuous path of commuting C^1 diffeomorphisms. The result extends to $\mathbb{Z}^k$ representations (i.e., k-tuples of commuting diffeomorphisms), but unfortunately, the diffeomorphisms in the connecting path are not shown to be more smooth than C^1.

This result was extended in [7] to show:

Theorem: The space of C^∞ orientation-preserving actions of $\mathbb{Z}^n$ on $[0,1]$ is connected. Similarly, the group of non-free actions of $\mathbb{Z}^2$ on the circle is connected.

Now let $\sum_g$ denote a compact surface with genus $g \geq 2$, and let $\Gamma_g = \pi_1(\sum_g, x_0)$ denote its fundamental group for some basepoint x_0.

The space of representations of Γ_g into $\mathrm{Diff}_+^r(\mathbb{S}^1)$ corresponds to the C^r-actions of Γ_g on the circle. The properties of these representation spaces have been extremely well-studied, and the topology of the representation variety is connected to the values of the euler class associated to the circle bundle over $\sum_g$ obtained [30, 61, 85]. Much less is known in the case of actions on the interval $[0,1]$. Let

$$\mathcal{R}(\Gamma_g, \mathrm{Diff}_+^\infty([0,1]))\} \;=\; \{\rho\colon \Gamma_g \to \mathrm{Diff}_+^\infty([0,1])\}$$

be the representation variety with the C^∞-topology.

PROBLEM 4.1. *What are the path-components of the space of representations $\mathcal{R}(\Gamma_g, \mathrm{Diff}_+^\infty([0,1]))\}$?*

PROBLEM 4.2. *Find invariants which distinguish the path components of $\mathcal{R}(\Gamma_g, \mathrm{Diff}_+^\infty([0,1]))\}$.*

5. Steve Hurder

5.1. **Exceptional minimal sets.** Let $\mathcal{F}$ be a C^r-foliation of a closed manifold M of codimension-q, for $r \geq 1$. Let $\mathfrak{M} \subset M$ be a minimal set for $\mathcal{F}$. That is, $\mathfrak{M}$ is a closed subset which is a union of leaves, and every leaf in $\mathfrak{M}$ is dense. We say that $\mathfrak{M}$ is *exceptional* if its intersection with every transversal to $\mathcal{F}$ is totally disconnected.

The construction of exceptional minimal sets for codimension-1 foliations by Sacksteder [79] and Rosenberg and Roussarie [75], along with Sacksteder's theorem that the Denjoy minimal set does not exist for C^2-foliations [80], can be viewed as the beginnings of the modern study of foliation dynamics. The works [10, 11, 45, 46, 58] all consider the structure of exceptional minimal sets in codimension-1. There remain open questions in this case, none-the-less, as discussed in the problem sets [39, 40].

However, the situation for higher codimension is essentially completely open.

PROBLEM 5.1. *Classify the exceptional minimal sets for codimension-q, for $q \geq 2$ and $r \geq 2$.*

In codimension-1, there is a dichotomy for the derivative of the holonomy maps on a minimal set, in that they either have slow (subexponential) growth, or exponential growth [79, 38, 44]. For higher codimensions, the transverse dynamics can be more complicated, as evidenced by the constructions of solenoidal minimal sets for flows of 3-manifolds, and hyperbolic minimal sets which have the transverse structure of a horseshoe. The work of Clark and Hurder [16] constructed solenoidal minimal sets for foliations with leaf dimensions at least 2, and this work has many references to prior works. A basic problem remains to find new constructions of such sets.

On the other hand, the work on the structure of laminations arising from tiling spaces has led to new topological classification results for exceptional minimal sets [14] and part of the question is to what extent does the fact that $\mathfrak{M}$ arises from a C^2-dynamical system restrict its topological type?

6. Victor Kleptsyn

6.1. **Anosov questions.** The following is the simplest case of a well-known existence problem:

PROBLEM 6.1. *Let M^4 be a simply connected closed 4-manifold. Show that there does not exist an Anosov diffeomorphism of M.*

The second problem was asked by Anosov in the 1960's. A *polynomial foliation* on $\mathbb{C}^2$ is defined by complex differential equations:

$$\begin{aligned} \dot{X} &= P_n(X,Y) \\ \dot{Y} &= Q_n(X,Y) \end{aligned}$$

where P_n, Q_n are homogeneous polynomials on degree n.

PROBLEM 6.2. *For the generic such polynomial foliation, is the generic leaf simply connected?*

7. Rémi Langevin

7.1. **Conformal geometry of foliations.** The Riemannian geometry of foliations studies properties of the ambient manifold M and the leaves of a foliation $\mathcal{F}$ on it, such as intrinsic and extrinsic invariants of the curvature for the leaves. In contrast, the conformal geometry of a foliation [51, 48] studies properties derived from properties of the geodesics, such as their intersections, and properties of pencils (families of geodesic rays based at a point) and evolutes of the leaves. Both of these approaches to the geometry of foliations can then be related to other properties of the foliation, such as the total curvature of leaves [6, 49, 50, 77], minimality for the leaves [78] and the mean curvature flow [93], the dynamics of the leafwise geodesic flow [91] and the foliation entropy defined in [26, 94].

In all of the works of this nature, it is helpful to have an ambient Riemannian metric on the manifold M such that the leaves of the foliation $\mathcal{F}$ have the "most simple" geometry. This suggest the very general (though vague) problem:

PROBLEM 7.1. *Let $\mathcal{F}$ be a C^∞ foliation on a closed manifold M. What is the simplest (or nicest) Riemannian metric on M which results in the simplest extrinsic geometry for the leaves?*

The study of Dupin and Canal foliations in [2, 51, 52] provide motivating examples for this question.

The following is a very old problem, that remains as attractive and inaccessible as always.

PROBLEM 7.2. *Let $\mathcal{F}$ be a codimension-one foliation of a closed oriented 3-manifold M. Give an interpretation of the Godbillon-Vey invariant $GV(\mathcal{F}) \in \mathbb{R}$ for $\mathcal{F}$ in terms of the conformal geometry of $\mathcal{F}$.*

Thurston gave an intuitive interpretation of $GV(\mathcal{F})$ in terms of the total "helical wobble" of the leaves [86, 74]. Gelfand offered an interpretation via the simplexes swept out by planes parallel to the leaves. Only the result of Reinhart and Wood [74] is precise, and not just an intuitive statement. But it is not a conformal invariant, as it is derived from the extrinsic properties of the immersions of the leaves. The problem is to find a precise calculation of the number $GV(\mathcal{F})$ in terms of conformal invariants, if possible.

8. Yoshifumi Matsuda

8.1. **Rotation numbers.** Let $\mathrm{Diff}^{\omega}_{+}(\mathbb{S}^1)$ denote the group of orientation-preserving, real analytic diffeomorphisms of the circle. For each $\varphi \in \mathrm{Diff}^{\omega}_{+}(\mathbb{S}^1)$, the Poincaré rotation number $\rho(\varphi) \in \mathbb{R}/\mathbb{Z}$. If φ has finite order, or more generally has a periodic orbit, then $\rho(\varphi) \in \mathbb{Q}/\mathbb{Z}$.

Let $\Gamma \subset \mathrm{Diff}^{\omega}_{+}(\mathbb{S}^1)$ be a finitely-generated subgroup. Let $\mathrm{rot}(\Gamma) \subset \mathbb{R}/\mathbb{Z}$ denote the set of Poincaré rotation numbers for the elements of Γ.

This work [57] studied the case when $\mathrm{rot}(\Gamma)$ is a finite set, and showed that if such a group is nondiscrete with respect to the C^1-topology then it has a finite orbit. This implies that if such a group has no finite orbit then each of its subgroups contains either a cyclic subgroup of finite index or a nonabelian free subgroup.

PROBLEM 8.1. *Suppose that* $\mathrm{rot}(\Gamma) \subset \mathbb{Q}/\mathbb{Z}$ *then must* $\mathrm{rot}(\Gamma)$ *be a finite set?*

This is known to be true if $\Gamma \subset \mathbf{PSL}(2, \mathbb{R})$ considered as acting on $\mathbb{S}^1$, which follows by Selberg's Lemma. But the results of Ghys-Sergiescu in [25] show that this is false, if we consider the embedding they obtain of the Thompson group $T \subset \mathrm{Diff}^{\infty}_{+}(\mathbb{S}^1)$, so that the analytic assumption is necessary.

PROBLEM 8.2. *Does* Γ *always contain a finite-index subgroup which is torsion-free?*

This is known to be true for subgroups $\Gamma \subset \mathbf{GL}(n, \mathbb{K})$ where $\mathbb{K}$ is a field with characteristic 0 by Selberg's Lemma. It is false for the Thompson group $T \subset \mathrm{Diff}^{\infty}_{+}(\mathbb{S}^1)$.

These two problems ask whether $\mathrm{Diff}^{\omega}_{+}(\mathbb{S}^1)$ is more similar to a finite dimensional Lie group, or whether it contains subgroups with properties of the Thompson group T as embedded in $\mathrm{Diff}^{\infty}_{+}(\mathbb{S}^1)$.

9. Gaël Meigniez

9.1. **Lie foliations modeled on** G**.** A smooth foliation $\mathcal{F}$ on a closed manifold M is said to be a Lie foliation modeled on a connected Lie group G [17, 21] if there exists a covering $\pi\colon \widetilde{M} \to M$ and a submersion $\Pi\colon \widetilde{M} \to G$ such that the lifted foliation $\widetilde{\mathcal{F}}$ under the covering map has leaves equal to the fibers of the map Π. A Lie foliation is necessarily a transversally complete Riemannian foliation, so the Molino structure theory for Riemannian foliations can be applied to obtain many results, as in [12, 27, 68, 69].

PROBLEM 9.1. *Classify the Lie foliations whose leaves have dimension two, and are covered by the hyperbolic plane* $\mathbb{H}^2$*.*

For example, one family of such foliations is obtained from taking a finitely-generated group Γ and a discrete, cocompact smooth action ρ on the product space $\mathbb{H}^2 \times G$, where the action is assumed to preserve the product structure, then $M \cong \mathbb{H}^2 \times G/\rho$ has a Lie G-foliation $\mathcal{F}_\rho$ induced from the product foliation on the covering $\mathbb{H}^2 \times G$.

10. Eva Miranda

10.1. **Symplectic foliations.** Consider a regular Poisson manifold M with a smooth codimension-one foliation $\mathcal{F}$ admitting a defining 1-form α. Using the existence of this 1-form, V. Guillemin and E. Miranda and A.R. Eva introduced in [29] two invariants in the leafwise cohomology groups: the Reeb class in leafwise cohomology $[c_{\mathcal{F}}] \in H^1(M, \mathcal{F})$ (cf. [36]), and a class $[\sigma_{\mathcal{F}}] \in H^2(M, \mathcal{F})$ derived from a global 2-form on M which restricts to leafwise symplectic forms.

Theorem: Let $\mathcal{F}$ be a regular Poisson manifold M with a smooth codimension-one foliation $\mathcal{F}$. Suppose that the invariants $[c_{\mathcal{F}}] = 0$ and $[\sigma_{\mathcal{F}}] = 0$, and $\mathcal{F}$ has a compact leaf, then M is a symplectic mapping torus.

PROBLEM 10.1. *Can we characterize the regular Poisson manifolds M with smooth codimension-one foliation $\mathcal{F}$ which have no compact leaf? Is there a type of rigidity result for such geometric structures?*

For example, an application of Tischler's Theorem [87, 54] and Ghys' work in [27] may prove useful.

11. Yoshuhiko Mitsumatsu

11.1. **Leafwise symplectic foliations on $\mathbb{S}^5$.** In a celebrated work [53], H. Blaine Lawson, Jr. constructed smooth codimension-one foliations on the 5-sphere $\mathbb{S}^5$. We quote from [64]:

> It was achieved by a beautiful combination of the complex and differential topologies and was a breakthrough in an early stage of the history of foliations. The foliation is composed of two components. One is a tubular neighbourhood of a 3- dimensional nil-manifold and the other one is, away from the boundary, foliated by Fermat-type cubic complex surfaces. As the common boundary leaf, there appears one of Kodaira-Thurston's 4-dimensional nil-manifolds. As each Fermat cubic leaf is spiraling to this boundary leaf, its end is diffeomorphic to a cyclic covering of Kodaira-Thurston's nil-manifold.

Meerssemann and Verjovsky in [66, 67] considered the existence of leafwise complex and symplectic structures on Lawsons foliations as well as on slightly modified ones. Then in [64] it was proved:

Theorem: Lawson's foliation on the 5-sphere $\mathbb{S}^5$ admits a leafwise symplectic structure.

For $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1$, the polynomial

$$f(Z_0, Z_1, Z_2) = Z_0^p + Z_1^q + Z_2^r + Z_0 Z_1 Z_2 = 0$$

defines a cusp singularity of the hypersurface at the origin in $\mathbb{C}^3$.

PROBLEM 11.1. *For the Milnor fibration of this singularity, does a construction similar to that in* [64] *work to produce a leafwise symplectic foliation on $\mathbb{S}^5$? Or at least, does the Milnor fibre admit an end-periodic symplectic structure?*

Note that the link is a *Solv* 3-manifold, hence it fibres over $\mathbb{S}^1$ and even fits into Mori's framework, so that an isotopic family of associated contact structure converges to the associated spinnable foliation.

If the answer to Problem 11.1 is "yes", can we further generalize it to higher dimensional case [65]?

PROBLEM 11.2. *Let F be the Fermat cubic surface. Does $F \times F$ admit an end-periodic symplectic structure? Does $F \times \mathbb{C}$ admit an end-periodic symplectic structure? In general, one can ask, if W and V are open symplectic manifolds which are periodic on their ends, does $W \times V$ admit such a structure also?*

11.2. **2-calibration and tautness.** A codimension-one foliated manifold $(M, \mathcal{F})$ with a closed 2-form ω which restricts to a symplectic form on each leaf is said to be *2-calibrated.* If M is a closed manifold, then $\mathcal{F}$ is necessarily taut.

PROBLEM 11.3. *Does there exist a 2-calibrated codimension-*1 *foliation on* $\mathbb{S}^5$*?*

A tangential symplectic structure on a foliation is the same thing as a *regular Poisson* structure.

PROBLEM 11.4. *Find a (classical) physical procedure to obtain a regular Poisson structure which can not be lifted to a global closed 2-form, that is, they are not 2-calibrated.*

PROBLEM 11.5. *Is any closed transversal to a taut codimension-*1 *foliation not null-homotopic? Is this true in the 2-calibrated case?*

PROBLEM 11.6. *Is it possible to destroy the compact leaf of Lawson's foliation without destroying a leafwise symplectic structure? (cf. Gaël Meigniez's h-principle* [60]*.)*

PROBLEM 11.7. *Does there exists an integral closed symplectic manifold* (W, ω) *of dimension* ≥ 4 *such that the associated unit* $\mathbb{S}^1$*-bundle of the pre-quantization admits a foliated bundle structure?*

The 2-dimensional case is well-known to have a positive solution. The construction of higher dimensional examples would provide explicit examples of the following result:

Theorem [Morita [71]] The universal euler class $e \in H^2(B\mathit{Diff}^\infty_+(\mathbb{S}^1); \mathbb{R})$ of flat $\mathbb{S}^1$-bundles has all its powers non-trivial. That is,

$$\forall n \in \mathbb{N}, \quad e^n \neq 0 \in H^{2n}(B\mathit{Diff}^\infty_+(\mathbb{S}^1); \mathbb{R}) \,.$$

There is no known explicit construction of a cycle (a foliated $\mathbb{S}^1$-bundle) with $e^n \neq 0$ for $n \geq 2$.

PROBLEM 11.8. *Find a (topological) obstruction to the existence of leafwise symplectic foliations.*

If the ambient closed manifold M has trivial $H^2(M, \mathbb{R})$, then the existence of a compact leaf for $\mathcal{F}$ is an easy obstruction. This is the situation for $M = \mathbb{S}^5$, for example.

11.3. **Moduli for leafwise complex structures.** For Lawson's foliation in [53] (the first of three simple elliptic hypersurface singularities), the boundary of the tube component is *Kodaira's primary surface* and admits a complex structure. Its moduli space is well understood.

Meerssemann and Verjovsky in [66, 67] analysed the moduli space of leafwise complex structures on foliated manifolds, and in particular for Lawson's foliation (see also G. Deschamps2010 [18]). They obtained the following results:

Theorem: The moduli space for the 3-dimensional Reeb component exactly coincides with that of the boundary torus through the restriction map.

Theorem: The tube component of Lawson's foliation does not admit a leafwise complex structure, that is, its moduli space is empty.

PROBLEM 11.9. *Find non-trivial examples of leafwise Kähler foliations. Conversely, find a (topological) obstruction to their existence.*

PROBLEM 11.10. *Prove that the tube component for a cusp singularity does not admit a leafwise complex structure.*

For the cusp singularities, first we should find a complex structure for the boundary. If it does not admit any, then already the problem is solved.

11.4. **Flat surface bundles.** Consider the representation space $\pi_0(\mathit{Hom}(\pi_1(\sum_g), \mathit{Diff}^\infty_+(\mathbb{S}^1)))$. Here are three problems about this space.

PROBLEM 11.11. *Assume that e divides $(2-2g) = \chi(\sum_g)$, and set $d = (2-2g)/e$. Consider the component of the representation variety which contains the d-fold covering of the Anosov foliation arising from the geodesic Anosov flow. Does it consist of topologically conjugate foliations?*

PROBLEM 11.12. *Let $\mathcal{F}_0$ and $\mathcal{F}_2$ be foliations associated to representations in different connected components, but with the same euler numbers. Does there exists a continuous family $\mathcal{F}_t$ for $0 \leq t \leq 1$ of taut foliations of the total space between $\mathcal{F}_0$ and $\mathcal{F}_1$?*

PROBLEM 11.13. *Let $\mathcal{F}_0$ and $\mathcal{F}_2$ be foliations associated to representations in different connected components. Does there exist a foliation $\mathcal{F}$ of codimension 2 on $W^4 = [0,1] \times M$ which is tangent to the boundary $\partial W = \{0,1\} \times M$ and restricts to $\mathcal{F}_i$ on $\{i\} \times M$ for $i = 0,1$? If so, give a geometric construction of $\mathcal{F}$.*

12. Hiraku Nozawa

12.1. **Secondary invariants.** Let $n \geq 3$ and (M, g_0) a compact hyperbolic n-manifold with constant sectional curvature. That is, we assume there is a uniform lattice $\Gamma \subset \mathrm{Isom}(\mathbb{H}^n)$ such that $M \cong \mathbb{H}^n/\Gamma$. Let g be a metric on M which is C^∞-close to a metric of constant negative curvature g_0 on M. Then the stable foliation $\mathcal{F}_g$ of the geodesic flow of g on the unit tangent sphere bundle SM of g_0 are $C^{1,\alpha}$ for $\alpha = 1 - \epsilon$ where $\epsilon < 1/n$ by results of Hasselblatt [34]. Thus, the extended Godbillon-Vey class $\mathrm{GV}(\mathcal{F}_g) \in H^{2n+1}(SM;\mathbb{R})$, defined as in [37], is well-defined.

PROBLEM 12.1. *Compute the Godbillon-Vey number* $\mathrm{GV}(\mathcal{F}_g)$ *of* $\mathcal{F}_g$.

In the case of $n = 2$, so-called Mitsumatsu Defect formula that computes $\mathrm{GV}(\mathcal{F}_g)$ was obtained by Mitsumatsu [62] and Hurder-Katok [37]. The answer of the problem will give an example of families of $C^{1,\alpha}$-foliations which are topological conjugate but have different Godbillon-Vey numbers in higher codimension, yet the foliations are topologically conjugate by the stability of Anosov flows.

13. Paul Schweitzer

13.1. **Space of foliations.** For a closed manifold N, and $I = [0,1]$, let $\mathrm{Fol}^r_c(N \times I)$ be the space of codimension-one, C^r-foliations on $N \times I$ with the C^r-topology for $1 \leq r \leq \infty$, and all leaves compact submanifolds, with the boundary being leaves as well. This space is easily seen to be locally contractible for $r = \infty$.

PROBLEM 13.1. *Show that* $\mathrm{Fol}^\infty_c(N \times I)$ *is connected, which is equivalent to it being path connected.*

PROBLEM 13.2. *Is each connected component of the space* $\mathrm{Fol}^\infty_c(N \times I)$ *contractible?*

Note that a closed manifold M has a compact codimension-one foliation, then some double covering of M it fibers over $\mathbb{S}^1$ with the leaves as fibers.

For the case $N = \mathbb{S}^2$ the space $\mathrm{Fol}^\infty(N \times I)$ is known to be connected, by Cerf's Theorem [13] and contractible by Hatcher's proof of the Smale Conjecture [35]. On the other hand, for some $3 \leq n \leq 5$, for the sphere $\mathbb{S}^n$, $\mathrm{Fol}^\infty(\mathbb{S}^n \times I)$ is not contractible, as a consequence of Burghelea's results on the homotopy types of diffeomorphism groups of compact manifolds.

PROBLEM 13.3. *Let $\sum_g$ be the closed oriented surface of genus $g \geq 1$. Show that* $\mathrm{Fol}^r_c(\sum_g \times [0,1])$ *is contractible, for $r \geq 1$.*

13.2. **Ends of leaves.** There is an extensive knowledge of the structure of codimension-one foliations on compact manifolds. However, some problems remain a just as much a puzzle as always. One general set of open questions concern the ends of non-compact leaves.

PROBLEM 13.4. *Must an isolated end of a non-compact leaf of a codimension-one foliation of a compact manifold be periodic?*

Here is a more specific version of this question:

PROBLEM 13.5. *Can a contractible* 3*-manifold that is not periodic at infinity occur as a such a leaf with isolated ends?*

For foliations with leaves of dimension 4, there is an analogous question, which has been asked at foliation meetings for many years. Recall that an exotic $\mathbb{R}^4$ is a smooth manifold X which is homeomorphic to Euclidean $\mathbb{R}^4$, but is not diffeomorphic to $\mathbb{R}^4$. There are in fact continuous families of such exotic beasts (see Gompf [31], Furuta and Ohta [24].) There are also constructions of continuous families of exotic differentiable structures of open 4-manifolds (see [5, 19]). All of these manifolds are not the coverings of a compact manifold, and it seems just as unlikely that they could be leaves of a foliation.

PROBLEM 13.6. *Show that an exotic $\mathbb{R}^4$ cannot be quasi-isometric to a leaf of a C^1-foliation of a compact manifold M in codimension-one.*

Though the conclusion seems very plausible, there is as yet no solution known. One supposes that the same sort of restriction is true for the other constructions of exotic open 4-manifolds, and also in higher codimension $q > 1$, but again, no results in this direction seem to be known.

14. Takashi Tsuboi

14.1. **Commutator width.** Let M^{2n} be a closed, even dimensional C^∞-manifold. It is known that the connected component $\mathrm{Diff}^\infty(M)_0$ of the identity of $\mathrm{Diff}^\infty(M)$ is a perfect group, so that every element $\phi \in \mathrm{Diff}^\infty(M)_0$ can be written as a product of commutators. The *commutator length* of ϕ is the least number of commutators required. The *commutator width* of $\mathrm{Diff}^\infty(M)_0$ is the uniform upper bound for all of the commutator lengths, which may possibly be infinite. For example, in the case $n = 2$ it is not known if the commutator width is finite.

PROBLEM 14.1. *Estimate the commutator width of* $\mathrm{Diff}^\infty(M)_0$.

It is known that the commutator width is finite for $2n \geq 6$ and may depend on the manifold M, as shown in [89]. It is also known that the commutator width is finite, and does not depend on M when M admits a handle decomposition without handles of the middle index [88].

A continuum is a non-empty, compact connected metric space $\mathfrak{M}$.

PROBLEM 14.2. *Characterize the continua $\mathfrak{M}$ such that every homeomorphism of $\mathfrak{M}$ is a commutator. That is, the commutator width of* $\mathrm{Homeo}(\mathfrak{M})$ *is one.*

For example, it is known that this is true if $\mathfrak{M} = \mathbb{S}^n$ (the group of orientation preserving homeomorphisms), or $\mathfrak{M}$ is a Menger space [90].

15. Vladimir Rovenski

15.1. **Totally geodesic foliations.** Let M^{n+p} be a connected manifold, endowed with a p-dimensional foliation $\mathcal{F}$, i.e., a partition of M into p-dimensional submanifolds. A foliation $\mathcal{F}$ on a Riemannian manifold (M, g) is *totally geodesic* if the leaves (of $\mathcal{F}$) are totally geodesic submanifolds. A Riemannian metric g on $(M, \mathcal{F})$ is called *totally geodesic* if $\mathcal{F}$ is totally geodesic with respect to g. We have the g-orthogonal decomposition $TM = D_{\mathcal{F}} \oplus D$, where the distribution $D_{\mathcal{F}}$ ($\dim D_{\mathcal{F}} = p$) is tangent to $\mathcal{F}$.

Let $\{e_i, \varepsilon_\alpha\}_{i\le n,\, \alpha\le p}$ be a local orthonormal frame on TM adapted to D and $D_{\mathcal{F}}$. The *mixed scalar curvature* is the following function on M, see [76], [92] etc:

$$\mathrm{Sc}_{\,\mathrm{mix}} = \sum\nolimits_{i=1}^{n} \sum\nolimits_{\alpha=1}^{p} K(e_i, \varepsilon_\alpha),$$

where $K(e_i, \varepsilon_\alpha)$ is the sectional curvature of the plane spanned by the vectors e_i and ε_α.

The integral formula with total $\mathrm{Sc}_{\,\mathrm{mix}}$, see [92], gives us decomposition criteria for foliations with an integrable orthogonal distribution D under the constraints on the sign of $\mathrm{Sc}_{\,\mathrm{mix}}$, for example:

(1) If $\mathcal{F}$ and $\mathcal{F}^\perp$ are complementary orthogonal totally umbilical and totally geodesic foliations on a closed oriented Riemannian manifold M with $\mathrm{Sc}_{\mathrm{mix}} \ge 0$, then M splits along the foliations.

(2) A compact minimal foliation $\mathcal{F}$ on a Riemannian manifold M with an integrable orthogonal distribution and $\mathrm{Sc}_{\,\mathrm{mix}} \ge 0$ splits along the foliations.

The basic question that we want to address is the following.

PROBLEM 15.1. *Which foliations admit a totally geodesic metric with* $\mathrm{Sc}_{\,\mathrm{mix}} > 0$*?*

Notice that a change of initial metric along orthogonal (to $\mathcal{F}$) distribution D preserves the property "$\mathcal{F}$ is totally geodesic". Let $\pi : M \to B$ be a fiber bundle with compact fibers. One may deform the metric g along D on a neighborhood of a fiber to obtain a bundle-like totally geodesic metric $\tilde g$ (which in general is not D-conformal to g) on the fiber. If there is a section $\xi : B \to M$ then the deformation can be done globally (on M), and π becomes a Riemannian submersion with totally geodesic fibers. In this case, the mixed sectional curvature is non-negative, see O'Neill's formula $K(X, V)|X|^2|V|^2 = |A_X V|^2$ ($X \in D$, $V \in D_{\mathcal{F}}$), moreover, if D is nowhere integrable then $\mathrm{Sc}_{\,\mathrm{mix}} > 0$ ($\mathrm{Sc}_{\,\mathrm{mix}} \equiv 0$ when D is integrable).

Due to above, we ask the following, which is a special case of Problem 15.2.

PROBLEM 15.2. *Given a Riemannian manifold (M, g) with a totally geodesic foliation $\mathcal{F}$, does there exist a D-conformal to g metric $\tilde g$ on M such that* $\mathrm{Sc}_{\,\mathrm{mix}} > 0$*?*

Problem 15.2 for Riemannian metrics on a fiber bundle was studied in [76], where it was shown:

Example: For any $n \ge 2$ and $p \ge 1$, there exists a fiber bundle with a closed $(n+p)$-dimensional total space and a compact p-dimensional fiber, having a totally geodesic metric of positive mixed scalar curvature.

To show this, consider the Hopf fibration $\tilde\pi\colon \mathbb{S}^3 \to \mathbb{S}^2$ of a unit sphere $\mathbb{S}^3$ by great circles (closed geodesics). Let $\tilde F$ and $\tilde B$ be closed Riemannian manifolds with dimensions, respectively, $(p-1)$ and $(n-2)$. Let $M = \tilde F \times \mathbb{S}^3 \times \tilde B$ be the metric product, and $B = \mathbb{S}^2 \times \tilde B$. Then $\pi : M \to B$ is a fibration with a totally geodesic fiber $F = \tilde F \times \mathbb{S}^1$. Certainly, $\mathrm{Sc}_{\,\mathrm{mix}} = 2 > 0$.

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