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Conjugation-Free Geometric Presentations Of Fundamental Groups Of Arrangements

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Abstract

The fundamental group of the complement is a very important topological invariant of line arrangements. It is very interesting to study the connection between the combinatorial invariants of the arrangement and its topological invariants.

For a line arrangement $\mathcal{L}$, one can define the graph $G(\mathcal{L})$ of its multiple points. Fan has proved that if this graph has no cycles, then the fundamental group of the complement is a direct sum of free groups and infinite cyclic groups. He has conjectured that the inverse implication is also correct. Recently, Eliyahu, Liberman, Schaps and Teicher proved this conjecture.

Hence, one can write the fundamental groups of these arrange- ments by means of generators and relations, where the generators are the topological ones, and the relations are of the following type:

$$x_{ik}x_{ik-1}\cdots x_{i1} = x_{ik-1}\cdots x_{i1}x_{ik} = \cdots = x_{i1}x_{ik}\cdots x_{i2},$$

where x_{ij} are the topological generators of the group without any conjugations. We call such a presentation a *conjugation-free geometric presentation*. The importance of this definition is that for arrangements whose fundamental group has a conjugation-free geometric presentation, one can easily compute the fundamental group by reading it directly from the arrangement without any computation.

We conjecture that the fundamental group of an arrangement $\mathcal{L}$ has a conjugation-free geometric presentation if its graph of mul- tiple points $G(\mathcal{L})$ is a $K4$-free graph.

The main result which we present in this talk is a first step in the way of proving the conjecture: An arrangement whose graph $G(\mathcal{L})$ is a cycle of length n has a conjugation-free geometric presentation of the fundamental group of the complement.

We also give the exact structure (by means of a semi-direct product) of the fundamental group in the case of a cycle of length 3, where all the multiple points are of multiplicity 3.

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