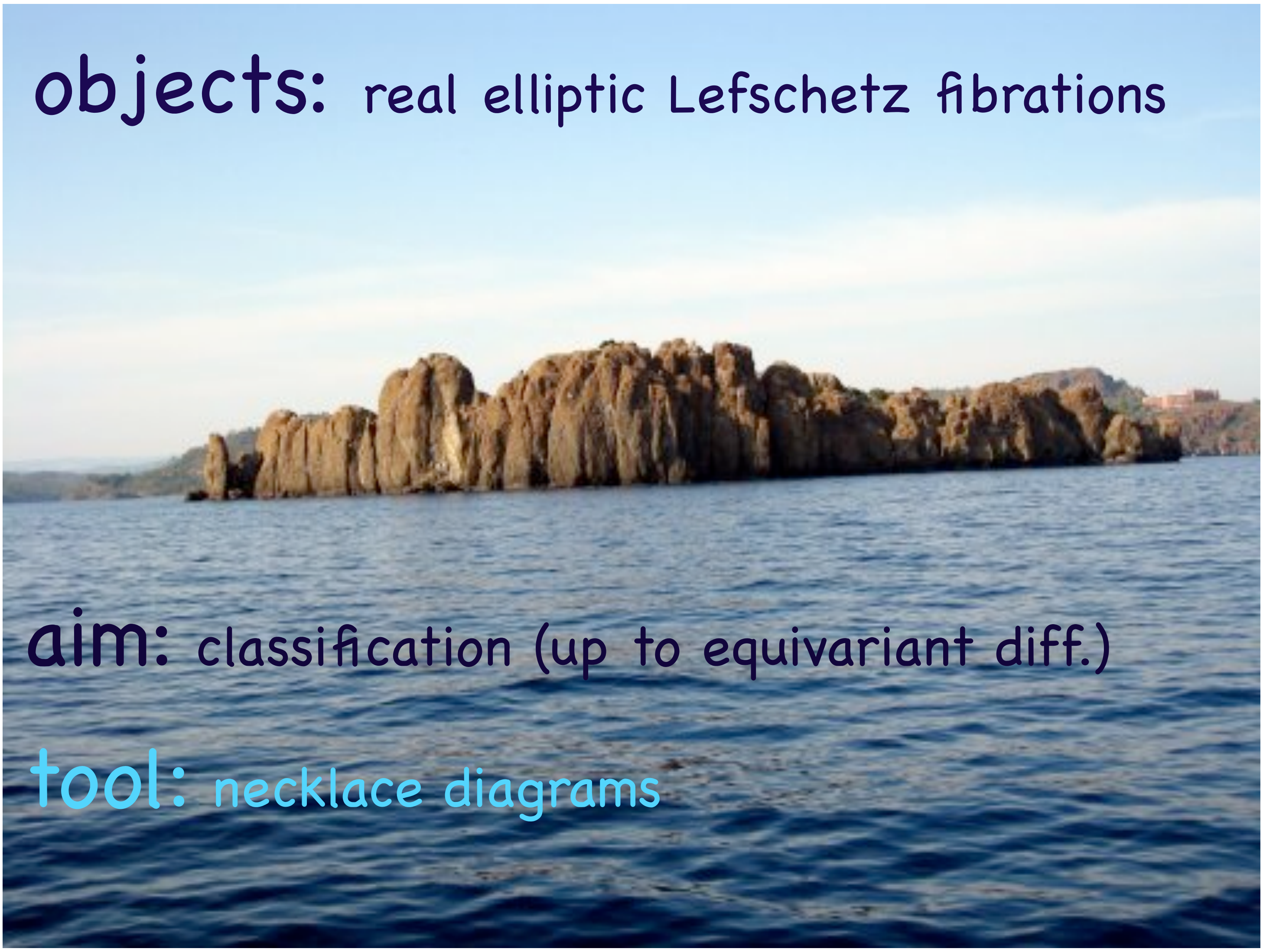
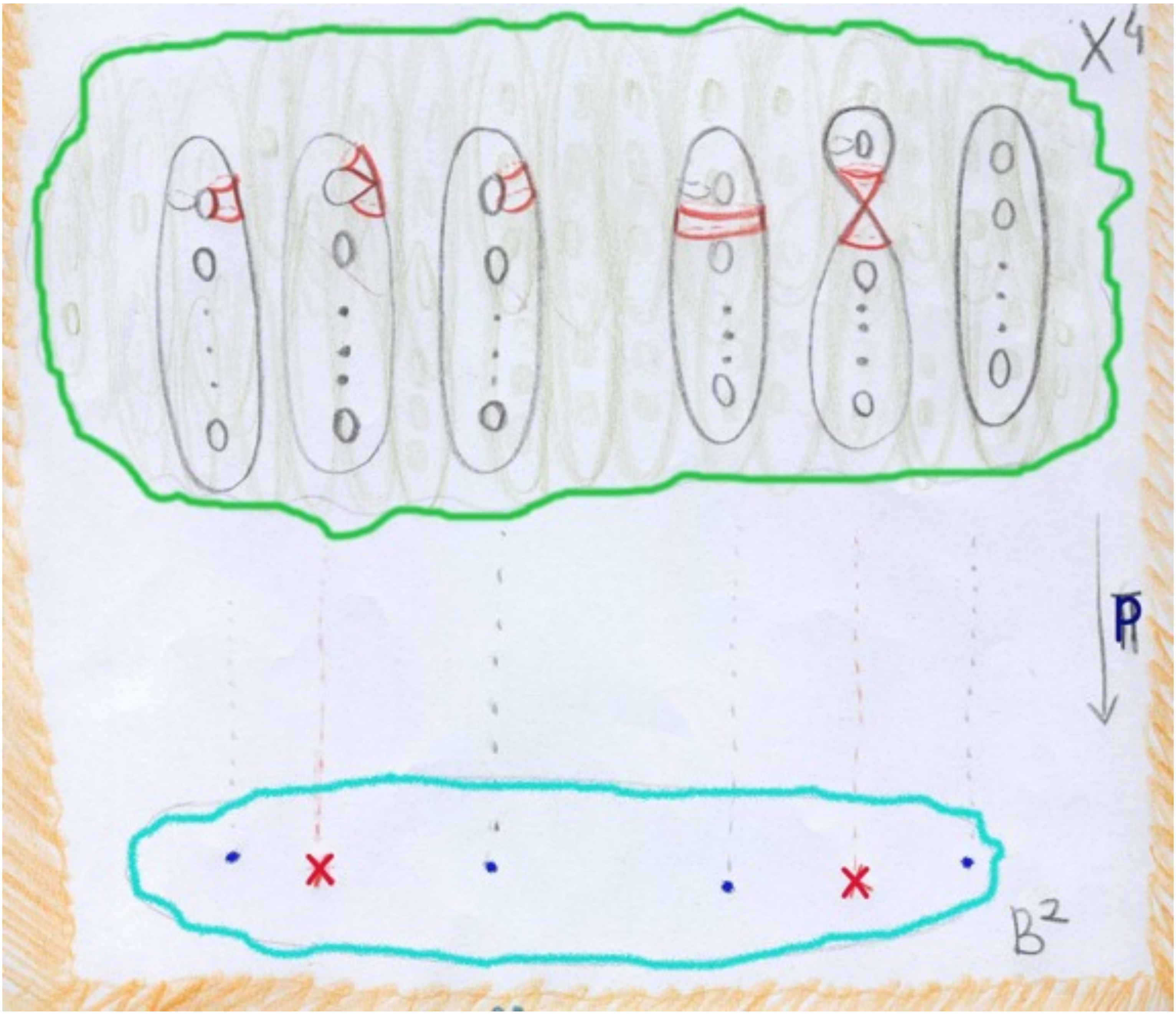

objects: real elliptic Lefschetz fibrations
aim: classification (up to equivariant diff.)
tool: necklace diagrams

(A) Lefschetz fibrations

$(p : X^4 \to B^2)$

"complex morse functions"

around critical points p looks like : $\mathbb{C}^2 \to \mathbb{C}$
$(z_1, z_2) \to z_1^2 + z_2^2$



(B) real structure

"smooth version of complex conjugation"

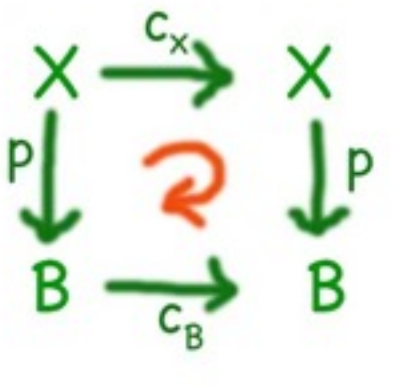
$c_X : X^4 \to X^4$

.orientation preserving involution
.dimension of fixed point set (if not empty) =2

$c_B : B^2 \to B^2$

.orientation reversing involution
(X, c): real manifold, Fix(c): real part

(C) real Lefschetz fibrations



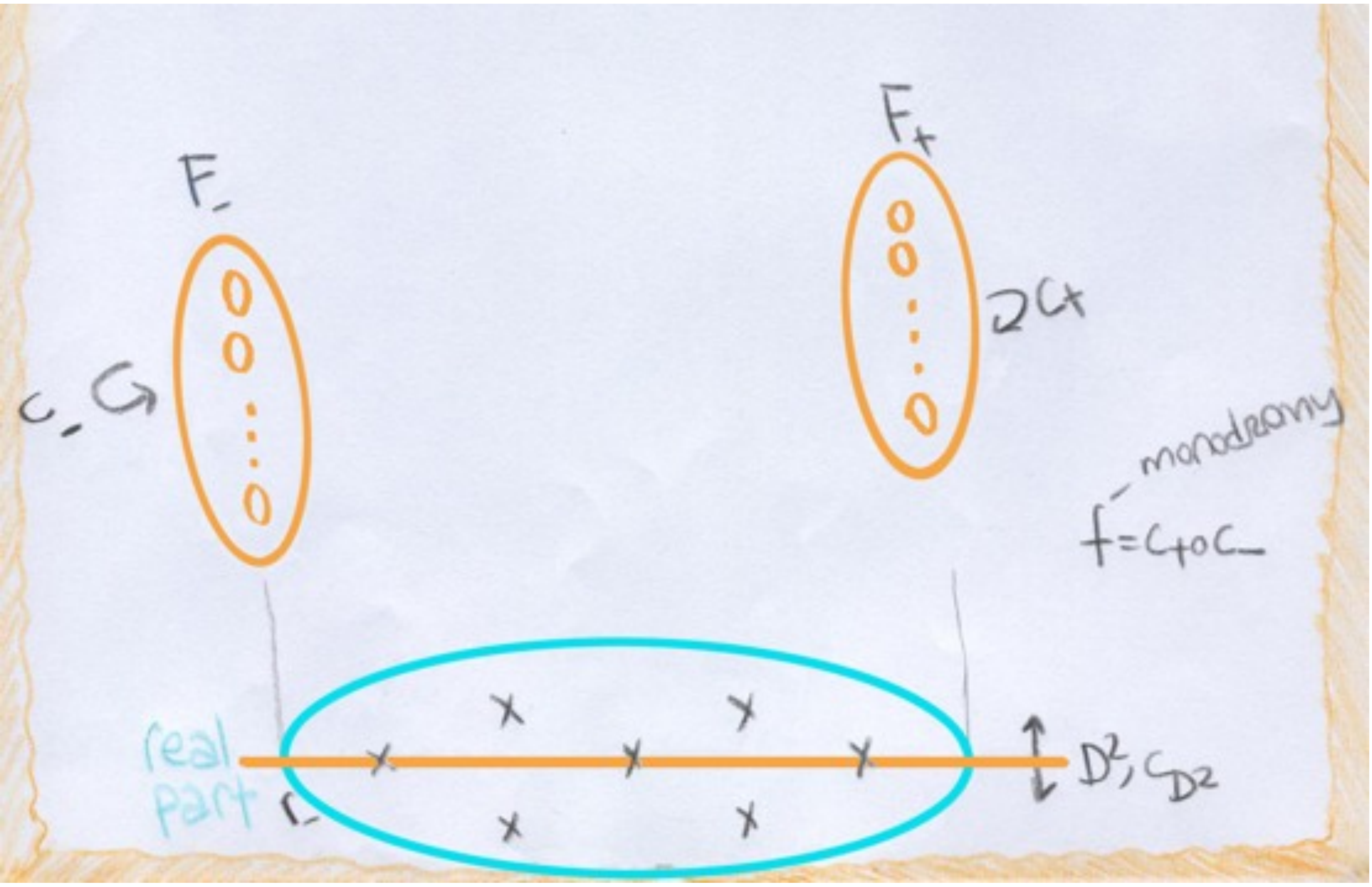
(D) elliptic: regular fiber

(E) some properties

1)critical sets are invariant under the action of real structures.

2)over real points of B, fibers inherit real structure from the real structure of X.

3)monodromy decomposes into product of two real structures.



(F) main theorem

1-1

*RELFs over sphere <---------> necklace diag.

up to symmetry

. have only real critical values

.monodromy=id

. admit a real section

1-1

REFINED

*RELFs over sphere <---------> necklace diag.

up to symmetry

. have only real critical values

.monodromy=id

(Moishezon & Livné, 1977)

1-1

*ELFs over sphere<----> # of critical values=12n

E(1)= $\mathbb{C}P^2 \# 9\mathbb{C}\bar{P}^2$

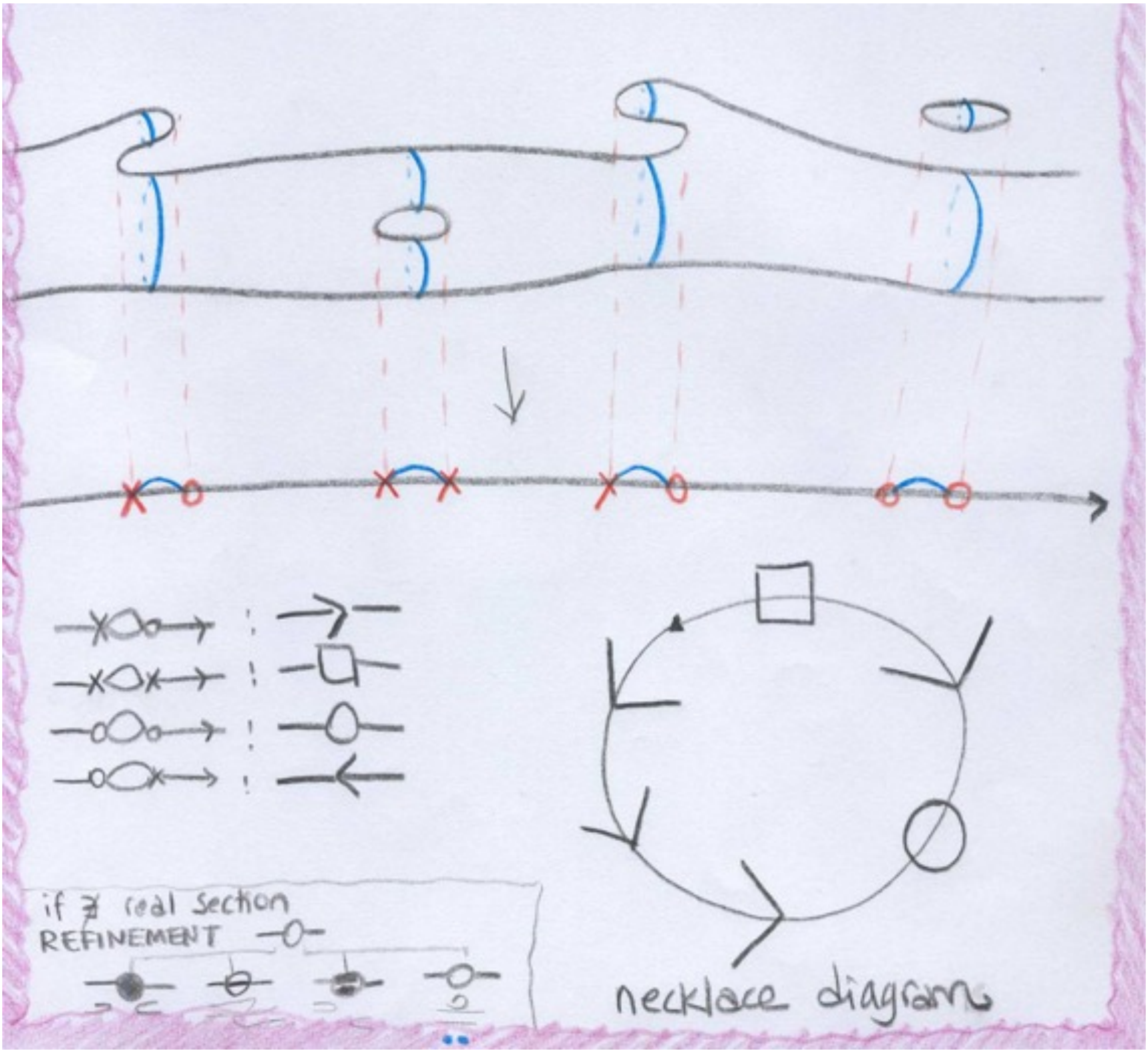
E(n)= E(n-1)# E(1)

(G) necklace diagrams

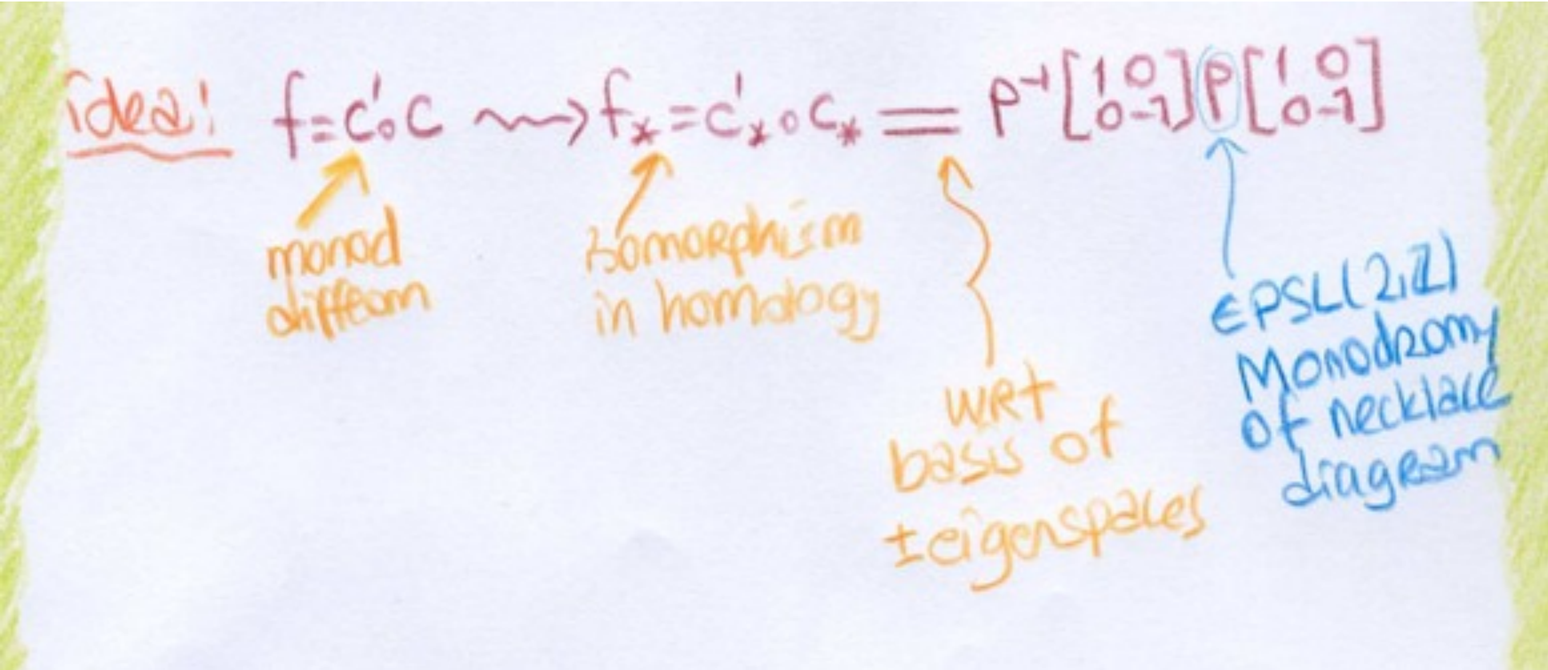
"from 4 to 2"

look at the real locus:

(assume for the moment that there exists a real section)

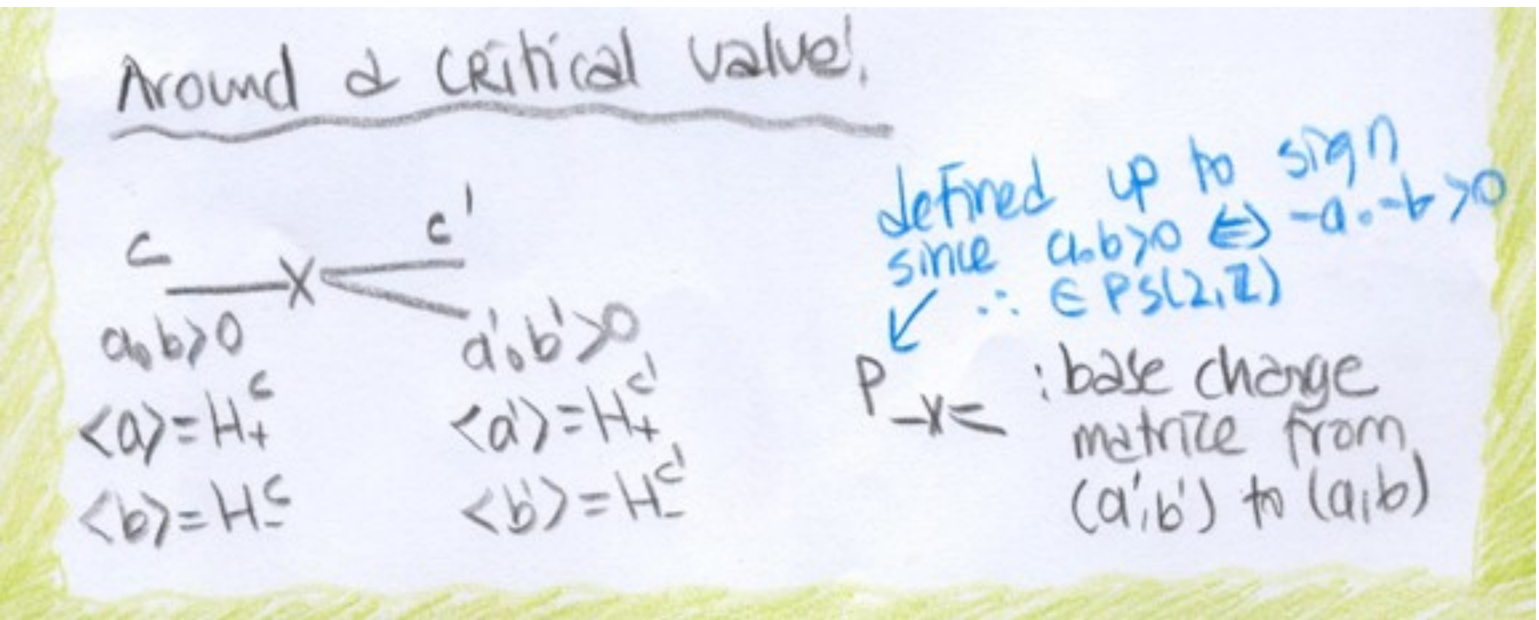


(H)monodromy of necklace diagrams



$$c : T^2 \to T^2 \Rightarrow c_* : H_1(T^2, \mathbb{Z}) \to H_1(T^2, \mathbb{Z})$$

$$H^c_\pm = \{a : c_*(a) = \pm a\}$$



$$PSL(2,\mathbb{Z}) = \{x, y : x^2 = y^3 = id\}$$

$$P_{-\circ <}P_{>\circ -} = xyxyx$$

$$P_{-\circ <}P_{>\times -} = xy^2$$

$$P_{-\times <}P_{>\circ -} = y^2x$$

$$P_{-\times <}P_{>\times -} = yxy$$

Necklace diagrams of real E(1) • having only real crit. values
• admitting a real section