

MCS 471: Formula Sheet for Exam I

1. Secant: $x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k)$, $k = 1, 2, \dots$

2. Newton: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$, $k = 0, 1, \dots$

or $x_{k+1} = x_k - m \frac{f(x_k)}{f'(x_k)}$, $k = 0, 1, \dots$, for $f(x) = (x - r)^m h(x)$, $h(r) \neq 0$.

3. Aitken: $x_k - \frac{(x_{k+1} - x_k)^2}{x_{k+2} - 2x_{k+1} + x_k}$, $k = 0, 1, \dots$

4. Golden Section: $x_1 = ca + (1 - c)b$, $x_2 = (1 - c)a + cb$, $c = \frac{-1 + \sqrt{5}}{2} \approx 0.6180$.

5. norms for $\mathbf{x} \in \mathbb{R}^n$:

$$\|\mathbf{x}\|_1 = \sum_{i=1}^n |x_i| \quad \|\mathbf{x}\|_\infty = \max_{i=1}^n |x_i| \quad \|\mathbf{x}\|_2 = \left(\sum_{i=1}^n x_i^2 \right)^{1/2}$$

6. norms for $A \in \mathbb{R}^{n \times m}$:

$$\|A\|_1 = \max_{j=1}^m \sum_{i=1}^n |a_{ij}| \quad \|A\|_\infty = \max_{i=1}^n \sum_{j=1}^m |a_{ij}|$$

$$\|A\|_f = \left(\sum_{i=1}^n \sum_{j=1}^m a_{ij}^2 \right)^{1/2} \quad \|A\| = \max_{\|\mathbf{x}\|=1} \|A\mathbf{x}\|$$

7. condition number $\text{cond}(A) = \|A\| \|A^{-1}\|$ for $A\mathbf{x} = \mathbf{b}$, $\mathbf{r} = \mathbf{b} - A\bar{\mathbf{x}}$:

$$\frac{\|\mathbf{r}\|}{\|\mathbf{b}\|} \frac{1}{\|A\| \|A^{-1}\|} \leq \frac{\|\mathbf{x} - \bar{\mathbf{x}}\|}{\|\mathbf{x}\|} \leq \|A\| \|A^{-1}\| \frac{\|\mathbf{r}\|}{\|\mathbf{b}\|}$$

$$\frac{\|\mathbf{x} - \bar{\mathbf{x}}\|}{\|\mathbf{x}\|} \leq \|A\| \|A^{-1}\| \frac{\|A - \bar{A}\|}{\|A\|}$$

for A : $A^T = A$, with $A\mathbf{v}^{(i)} = \lambda_i \mathbf{v}^{(i)}$, and $|\lambda_1| \geq |\lambda_2| \geq \dots \geq |\lambda_n|$:

$$\|A\|_2 = |\lambda_1| \quad \|A^{-1}\|_2 = |\lambda_n^{-1}| \quad \|A\|_2 \|A^{-1}\|_2 = \left| \frac{\lambda_1}{\lambda_n} \right|$$

8. Newton for $f_1(\mathbf{x}) = 0$, $f_2(\mathbf{x}) = 0$, \dots , $f_n(\mathbf{x}) = 0$:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \Delta \mathbf{x} = - \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_n(\mathbf{x}) \end{bmatrix}, \quad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}, \quad k = 0, 1, \dots$$