MCS 471: Formula Sheet for Exam II

1. Lagrange interpolation: $l_i(x) = \prod_{\substack{j=0 \\ \neq i}}^{n} \frac{x - x_j}{x_i - x_j}, \quad p(x) = \sum_{i=0}^{n} l_i(x) f_i.$

2. Neville interpolation: $p_{i \ldots j} = \frac{(x^* - x_j) p_{i \ldots j-1} - (x^* - x_i) p_{i+1 \ldots j}}{x_i - x_j}.$

3. Divided differences: $f_{0 \ldots ji} = \frac{f_{0 \ldots j-1j} - f_{0 \ldots j-1i}}{x_j - x_i}$
   $p(x) = f_0 + f_{01}(x - x_0) + f_{012}(x - x_0)(x - x_1) + \cdots + f_{012 \ldots n}(x - x_0)(x - x_1) \cdots (x - x_{n-1}).$

4. Interpolation error: $E(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_1) \cdots (x - x_n).$

5. Chebyshev polynomials: $T_n(x) = \cos(n \arccos(x))$
   $T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2x T_n(x) - T_{n-1}(x), \quad n > 0.$

6. $Df(x) = \frac{\partial f}{\partial x}, \quad \Delta f(x) = \frac{f(x+h) - f(x)}{h}, \quad \nabla f(x) = \frac{f(x) - f(x-h)}{h}, \quad \delta f(x) = \frac{f(x+h) - f(x-h)}{2h}.$

7. Richardson extrapolation: $\Delta f(x, h, rh, \ldots, r^n h) = \frac{\Delta f(x, h, rh, \ldots, r^{n-1} h) r^n - \Delta f(x, rh, r^2 h, \ldots, r^n h)}{r^n - 1},$
   $\Delta f(x, h) = \Delta f(x), \; \delta f(x, h) = \delta f(x), \; \delta f(x, h, rh, \ldots, r^n h) = \frac{\delta f(x, h, rh, \ldots, r^{n-1} h) r^{2n} - \delta f(x, rh, r^2 h, \ldots, r^n h)}{r^{2n} - 1}.$

8. Trapezoidal rule: $\int_a^b f(x) dx = \frac{f(a) + f(b)}{2}(b - a),$
   composite Trapezoidal rule: $T(h) = \frac{h}{2}(f(a) + f(b)) + h \sum_{k=1}^{n-1} f(a + kh), \quad h = \frac{b-a}{n}.$
   Romberg integration: $T[i][j] = \frac{T[i][j-1] 2^{2j} - T[i-1][j-1]}{2^{2j} - 1}, \quad T[i][0] = T\left(\frac{h}{2^i}\right).$

9. Fourier series: $f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(2\pi kt) + b_k \sin(2\pi kt) = \sum_{k=-\infty}^{\infty} c_k e^{i 2\pi kt}.$
   $a_k = \int_0^1 f(t) \cos(2\pi kt) dt, b_k = \int_0^1 f(t) \sin(2\pi kt) dt, \quad c_k = \frac{1}{2}(a_k - i b_k), c_{-k} = \frac{1}{2}(a_k + i b_k).$