MCS 471: Formula Sheet for Chapters 6 and 7

1. Euler's method: $y_{n+1} = y_n + hf(x_n, y_n)$ to solve $\frac{dy}{dx} = f(x, y(x))$
   and the modified Euler's method: $y_{n+1} = y_n + \frac{h}{2}\left(f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right)$.

2. A third-order Runge-Kutta formula to solve $\frac{dy}{dx} = f(x, y(x))$:

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1) \\ k_3 &= hf(x_n + \tfrac{3}{2}h, y_n + \tfrac{3}{4}k_2) \\ y_{n+1} &= y_n + \tfrac{1}{9}(2k_1 + 3k_2 + 4k_3) \end{aligned}$$

3. A fourth-order Runge-Kutta formula to solve $\frac{dy}{dx} = f(x, y(x))$:

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1) \\ k_3 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_2) \\ k_4 &= hf(x_n + h, y_n + k_3) \\ y_{n+1} &= y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

4. Some Adams-Bashforth formulas to solve $\frac{dy}{dx} = f(x, y(x))$:

$$\begin{aligned} y_{n+1} &= y_n + \tfrac{1}{2}h(-f_{n-1} + 3f_n) \\ y_{n+1} &= y_n + \tfrac{1}{12}h(5f_{n-2} - 16f_{n-1} + 23f_n) \\ y_{n+1} &= y_n + \tfrac{1}{24}h(9f_{n-3} + 37f_{n-2} - 59f_{n-1} + 55f_n) \\ y_{n+1} &= y_n + \tfrac{1}{720}h(251f_{n-4} - 1274f_{n-3} + 2616f_{n-2} - 2774f_{n-1} + 1901f_n) \\ y_{n+1} &= y_n + \tfrac{1}{1440}h(-475f_{n-5} + 2877f_{n-4} - 7298f_{n-3} + 9982f_{n-2} - 7923f_{n-1} + 4277f_n) \end{aligned}$$

5. Some Adams-Moulton formulas to solve $\frac{dy}{dx} = f(x, y(x))$:

$$\begin{aligned} y_{n+1} &= y_n + \tfrac{1}{2}h(f_n + f_{n+1}) \\ y_{n+1} &= y_n + \tfrac{1}{12}h(-f_{n-1} + 8f_n + 5f_{n+1}) \\ y_{n+1} &= y_n + \tfrac{1}{24}h(f_{n-2} - 5f_{n-1} + 19f_n + 9f_{n+1}) \\ y_{n+1} &= y_n + \tfrac{1}{720}h(-19f_{n-3}106f_{n-2} - 264f_{n-1} + 646f_n + 251f_{n+1}) \\ y_{n+1} &= y_n + \tfrac{1}{1440}h(27f_{n-4} - 173f_{n-3}482f_{n-2} - 798f_{n-1} + 1427f_n + 475f_{n+1}) \end{aligned}$$

6. A central-difference approximation: $f''(x_i) = \frac{f(x_i+h) - 2f(x_i) + f(x_i-h)}{h^2} + O(h^2)$, $h > 0$.