MCS 471 Project One : Numerical Evaluation of Rational Expressions

The goal of this project is to investigate the numerical evaluation of rational expressions. In this project we will use Maple 7 (older versions of Maple will probably work as well) because one interesting feature about Maple is that we can set the working precision to any arbitrary number of digits.

0. Random Rational Functions

We will generate a quotient of two random polynomials. The command **randpoly** in Maple generates a polynomial of degree five with integer coefficients between -99 and +99. We will use this command to generate a rational function:

```
> randomize():                      # randomize seed
> p := randpoly(x);                 # numerator
> q := randpoly(x);                 # denominator
> r := p/q: rf := unapply(r,x);     # rational function
```

1. Evaluation at Random Points

We evaluate the rational function at random points, using floating-point arithmetic. We compare with the exact answer.

```
> Digits := 4;                      # working precision is four digits
> x0 := stats[random,normald](1);   # random point
> rfx0 := rf(x0);                   # approximate function evaluation
> x1 := convert(x0,'rational');     # rational representation of point
> rfx1 := rf(x1);                   # exact function evaluation
> err := abs(rfx0-rfx1);            # error
```

Assignment One: Repeat the calculation above for increasing values for Digits. Make a table with the error between the approximate and exact function evaluation for these values of Digits. Take enough (at least four) values for Digits until you see the general relation between precision (number of digits) and the error for arbitrary large values of Digits. What is this general relation?

2. Evaluation at a Special Points

Now we generate a special point and compare again the approximate function value with the exact one. Consider the following experiment:

```
> Digits := 4;                      # working precision is four digits
> x0 := fsolve(q,x);                # a special point
> x1 := convert(x0,'rational');     # rational representation of point
> rf(x1);                           # exact function evaluation
```

Assignment Two: Repeat the experiment above for increasing values for Digits. Make a table for values for rf(x1) for all values for Digits you choose. Again, take enough (at least four) values for Digits until you see a general pattern emerging. What happens to `rf(x1)` as you let Digits grow larger and larger? Is this what you expect?

3. Sensitivity Analysis

In the first experiment we took a random point and compared the result obtained with floating-point arithmetic with the result obtained with exact arithmetic. In the second experiment we took the rational

representation of a special point and computed the exact function evaluation. We repeated this experiment with increasing precision of the special point.

Assignment Three: In the experiments above we know both the random and special point only up to a limited number of digits. So in both cases there is an error of magnitude $10^{-Digits}$ on the points. What is the influence of such an error on the function evaluation in these points? In particular answer the following questions:

1. Given an error of 10^{-8} on a random point, what is the error on the function value? Do an experiment with enough digits to illustrate your answer.
2. Given an error of 10^{-8} on the special point, what is the error on the function value? Do an experiment with enough digits to illustrate your answer.
3. Is there a difference in your answer between the random and special point? Explain why there is or why there is no difference.

4. Deadline is Friday 25 January, at 1PM

Bring your project solution to class. It should contain the following:

1. The tables with numerical values for the experiments you have done.
2. Answers to the questions in the assignments. Please write complete grammatically correct sentences.
3. A print out of the Maple worksheet with your experiments.

See http://www.math.uic.edu/~jan/MCS471 for the hypertext version of this project and for the companion worksheet. Your are advised to download that worksheet, because it provides a good start for the project.

If you have questions, comments, or difficulties, feel free to come to my office for help.