Introduction to Symbolic Computation: a Maple/MATLAB Course

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Maple Conference 2005, 17-21 July
Wilfried Laurier University, Waterloo Ontario, Canada

Outline of the Talk

1. Symbolic Computation in the Curriculum
2. Goals of the Course
3. List of Topics
4. Computer Projects
5. Textbook and Lecture Notes

This material is based upon work supported by the National Science Foundation under Grant No. 0134611.
Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Computer Algebra in Undergraduate Education

R.M. Corless and D.J. Jeffrey: Scientific computing: One part of the revolution.
J. Symbolic Computation, 23(5-6):485–495, 1997.

E. Kaltofen: Teaching computational abstract algebra.
J. Symbolic Computation, 23(5-6):503–515, 1997.

M.B. Monagan: Worksheets and notebooks: Can we teach mathematical algorithms with them?
J. Symbolic Computation, 23(5-6):535–549, 1997.

Computational Mathematics at UIC

MCS = *Mathematical Computer Science*

contains Computational Mathematics

undergrad: MCS 320 **Introduction to Symbolic Computation**

→ systematic introduction to Maple and MATLAB

serves MCS 471 Numerical Analysis

→ focus on algorithms and computer projects

serves MCS 472 Introduction to Industrial Math & Computation

→ modeling and applications

graduate: MCS 563 Analytic Symbolic Computation

→ symbolic-numeric algorithms to solve polynomial systems

Symbolic Computation is ***as important as*** Numerical Analysis!

Goals of the Course

empower students to use scientific software

High level of Scientific Programming:

a Maple worksheet is a high level mathematical program.

Study of problems in Symbolic Computation:

just as numerical analysis is an important field of study.

Integration of Symbolic and Numeric Tools:

solving practical applications requires a good combination.

Students should not wait till graduate school for a systematic and structured way to the major software tools of scientific computing.

List of Topics

one semester course of 15 weeks, we meet three times a week

1. **First Steps with Maple:** (9 lectures)
   proper use of worksheets and extended arithmetic.
2. **Polynomials and Rational Expressions:** (5 lectures)
   data representation and expression manipulation.
3. **Calculus:** (6 lectures) functions and procedures, remember tables for recursion, symbolic and automatic differentiation.
4. **Advanced Maple:** (8 lectures) composite data structures, plotting; solving polynomial, differential and linear equations.
5. **Introduction to MATLAB:** (9 lectures)
   still the computing standard among engineers ...

course offered four times by the author, once by Anton Leykin

Computer Projects

three projects are assigned during the semester

1. use Maple's **extended arithmetic**
   → coding and cryptography is an excellent source
   e.g.: Chinese Remainder Theorem in Cryptography

2. graphing, use of difference-differential equations
   → **popular topics in applied mathematics**
   e.g.: Simple Model of Billiards

3. MATLAB is harder to use!
   e.g.: Wire-frame modeling with MATLAB

projects are worth 200 of the 700 points

Some Sources of Inspiration

R.H. Enns and G.C. McGuire: Computer Algebra Recipes: A Gourmet's Guide to the Mathematical Models of Science. Springer-Verlag, 2002.

W. Gander and J. Hřebíček: Solving Problems in Scientific Computing Using Maple and MATLAB. Springer-Verlag, third edition, 1997.

R.E. Klima, N. Sigmon, and E. Stitzinger: Applications of Abstract Algebra with Maple. CRC Press, 2000.

Sketches of Final Exam Questions

- Draw the Directed Acyclic Graph from the output of the dismantle command on an expression.
- Given a three terms recursion formula, write an efficient recursive indexed procedure.
- To draw a planar curve with singularity at $(0, 0)$, convert to polar coordinates.
- Give examples of exact, symbolic, and numeric factorizations of multivariate polynomials. Describe differences and similarities.
- Compute the distance to a surface with Lagrange multipliers using a Gröbner basis.

hands-on lectures, hands-on exam

Textbook and Lecture Notes

André Heck: **Introduction to Maple**.
Springer-Verlag, second edition, 1996.

– ran out of print before 3rd edition available;

– too difficult for our undergraduate students.

Lecture notes developed from Maple worksheets, available online.

`http://www.math.uic.edu/~jan/mcs320.html`

Plan to evolve into an eBook, to keep its contents current.

Discussion and Perspectives

- teaching mathematics from *a technological point of view*:
  e.g.: launching the Gröbner engine to solve a problem
- symbolic computation is an important discipline in its own right, merits its place in the undergraduate curriculum, just like numerical analysis
- appeal to those – both inside and outside computer algebra – interested in renewing how we teach mathematics, to develop introductory courses in symbolic computation