SI2-SSE: Solving Polynomial Systems with PHCpack and phcpy

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PHCpack and phcpy

PHCpack is a package for Polynomial Homotopy Continuation
to solve polynomial systems. ACM Transactions on Mathematical Software
achived version 1.0 as Algorithm 795, vol. 25, no. 2, pages 251–276, 1999.

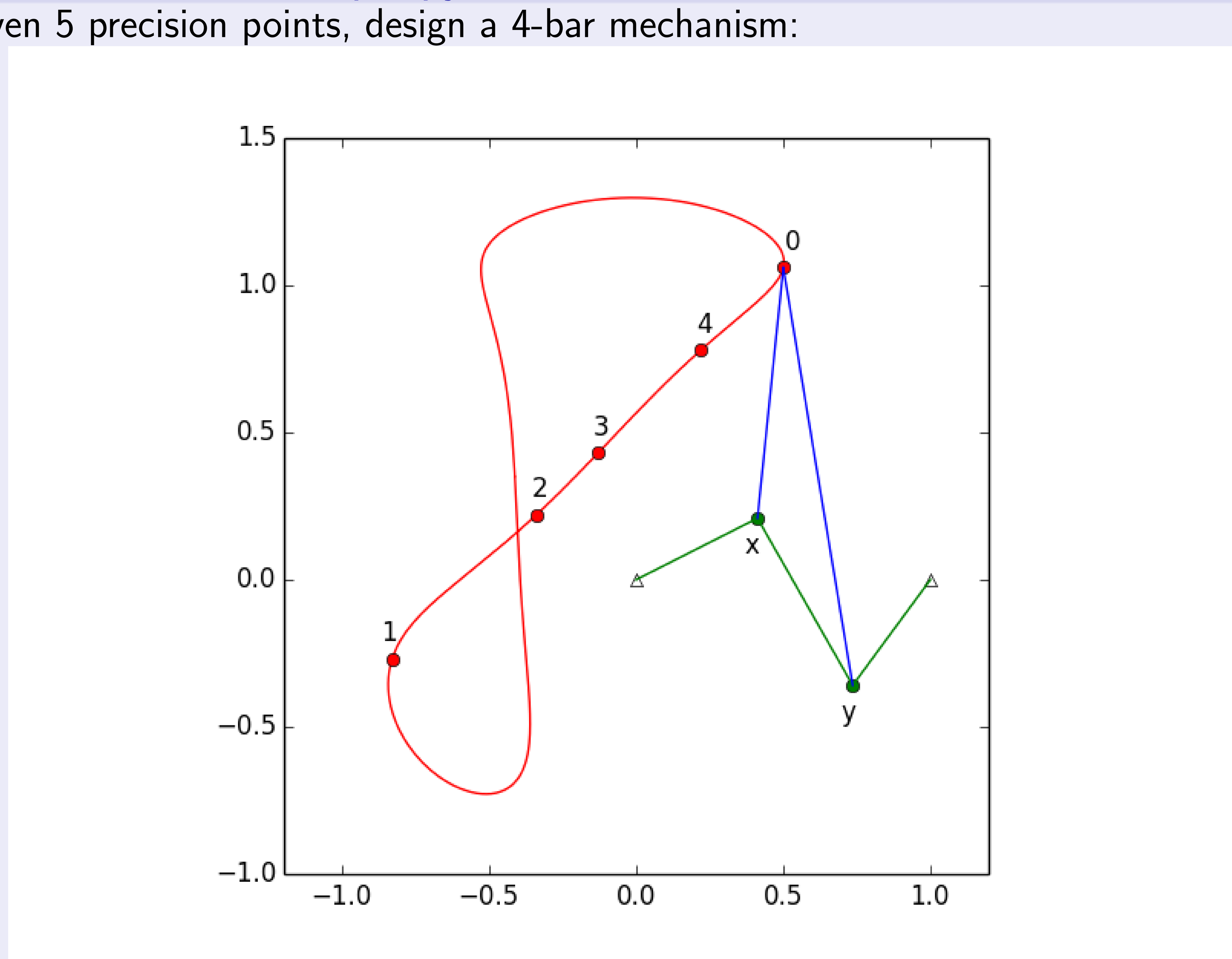
• Two *blackbox solvers:*

`phc -b` approximates all isolated solutions of a polynomial system.
`phc -B` computes a numerical irreducible decomposition, that is:
all positive dimensional solution sets, separated from the isolated solutions.
phcpy is a new Python package to export the functionality of PHCpack.

• Distributed under the GNU General Public License. Source code at
at `https://github.com/janverschelde/PHCpack`.

a use case from the phcpy tutorial

Given 5 precision points, design a 4-bar mechanism:



Reproduces a picture in a paper published in the Journal of Mechanical Design.

contributors

Four research assistants contributed to the project:

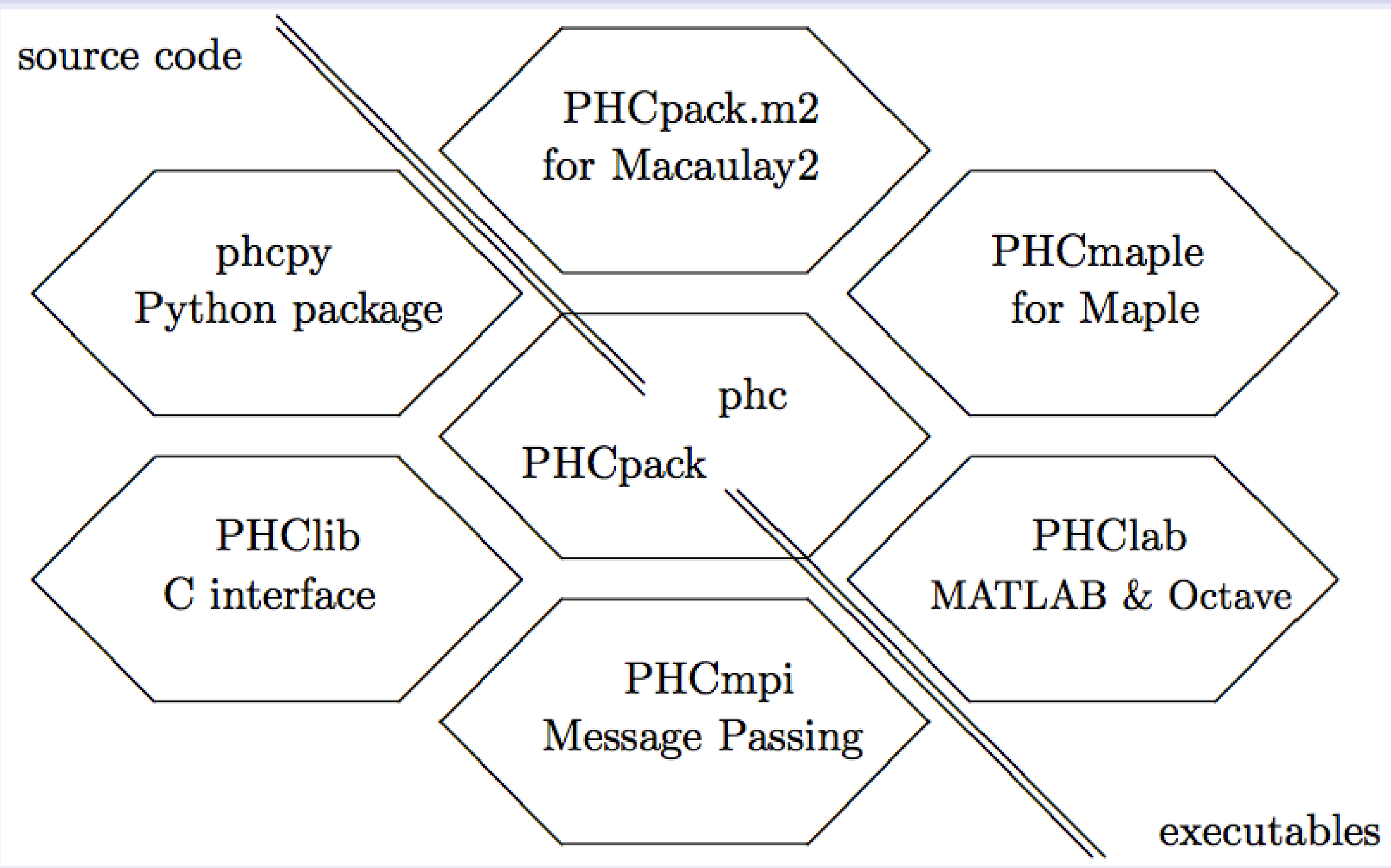
• Xiangcheng Yu (PhD 2015) wrote code for the web server
and GPU accelerated path trackers.

• Nathan Bliss (PhD 2018) designed algorithms to compute power series for
solution paths defined by polynomial homotopies.

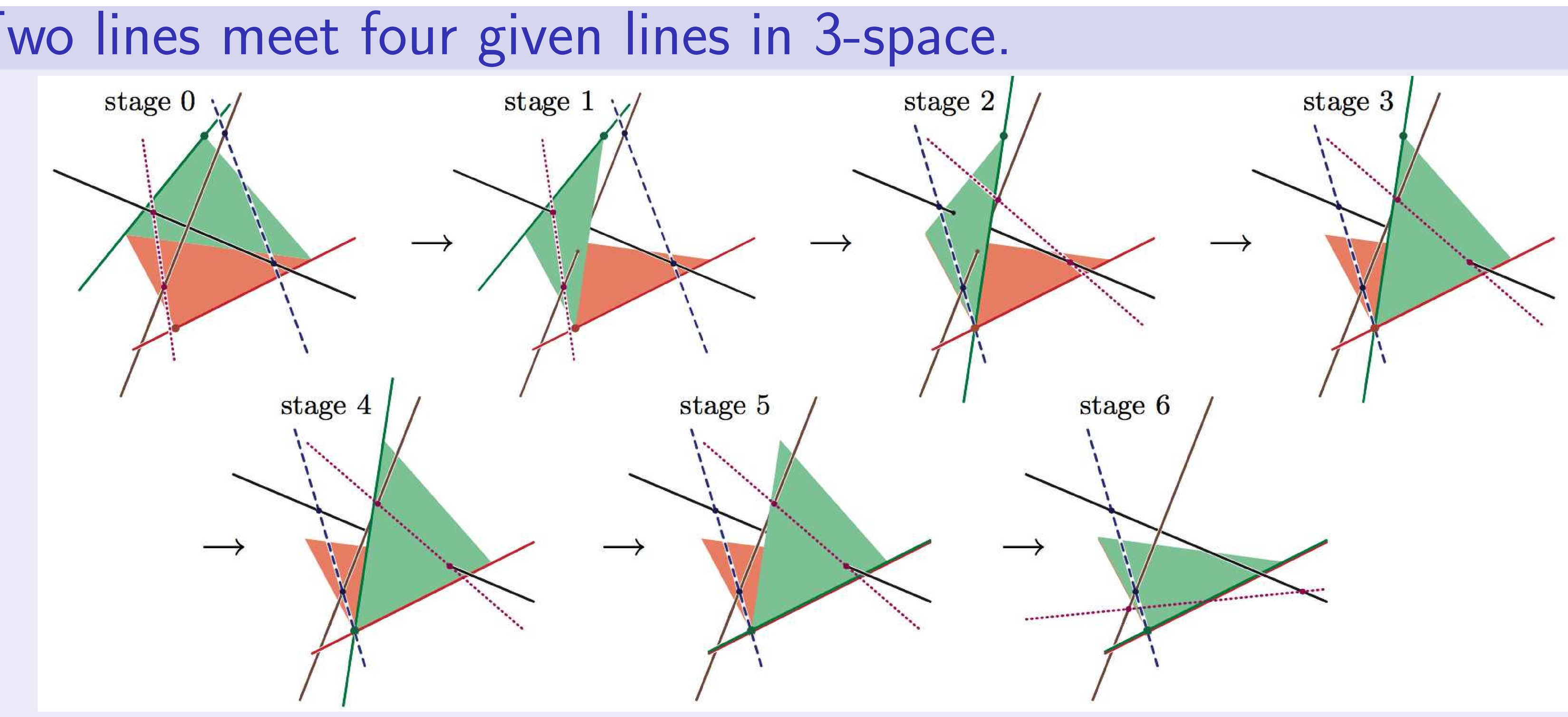
• Jeff Sommars (PhD 2018) developed parallel algorithms to compute tropical
prevarieties, with application to the cyclic 16-roots problem.

• Jasmine Otto helped deploy JupyterHub at `http://www.phcpack.org`.

interfaces

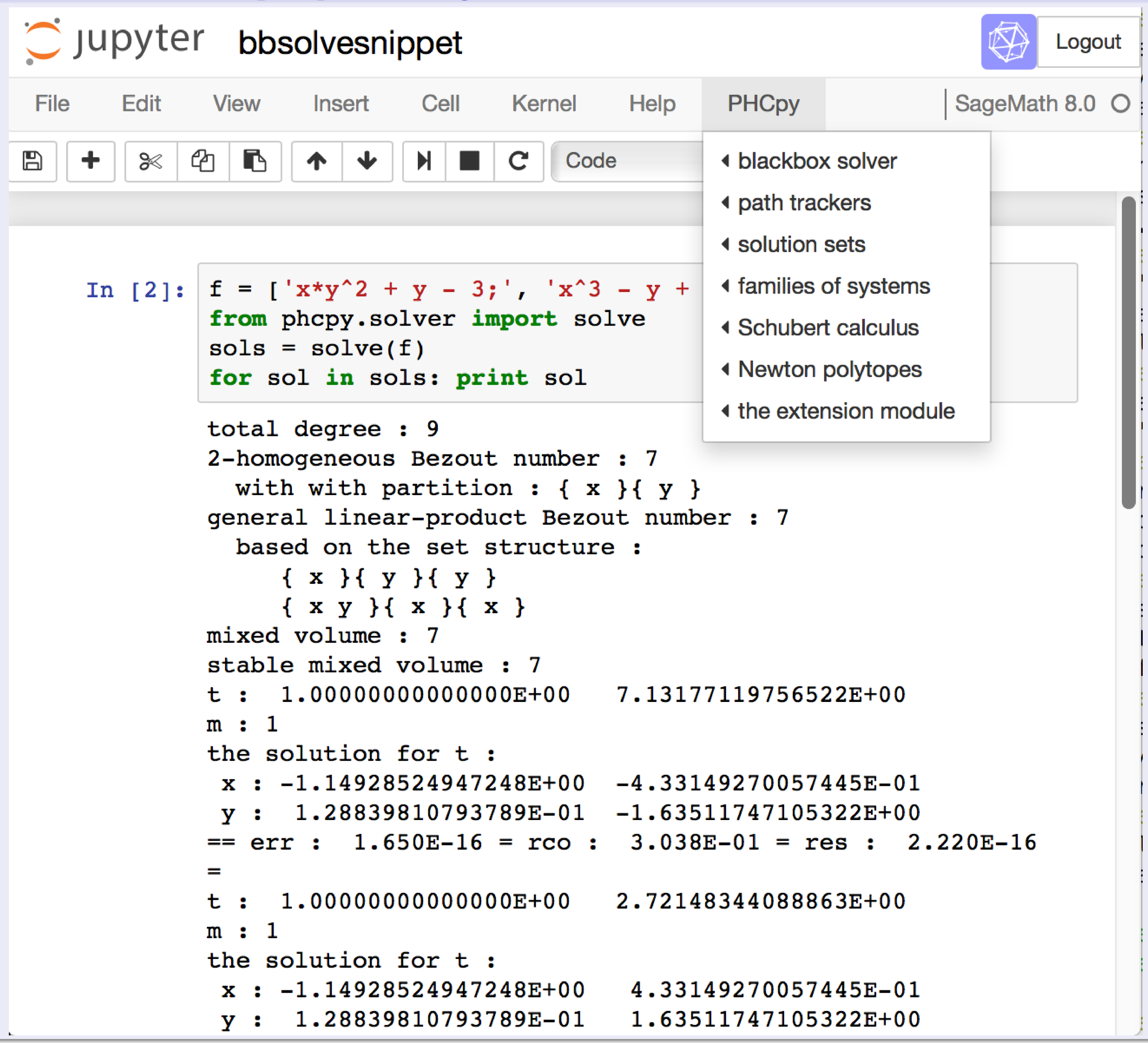


Two lines meet four given lines in 3-space.



A. Leykin, A. Martin del Campo, F. Sottile, R. Vakil, and J. Verschelde:
Numerical Schubert Calculus via the Littlewood-Richardson Homotopy Algorithm
`arXiv:1802.00984`.

available at `www.phcpack.org`



parallelism

Almost all computers have multicore processors.
Graphic cards have surpassed ordinary CPUs in computing power.

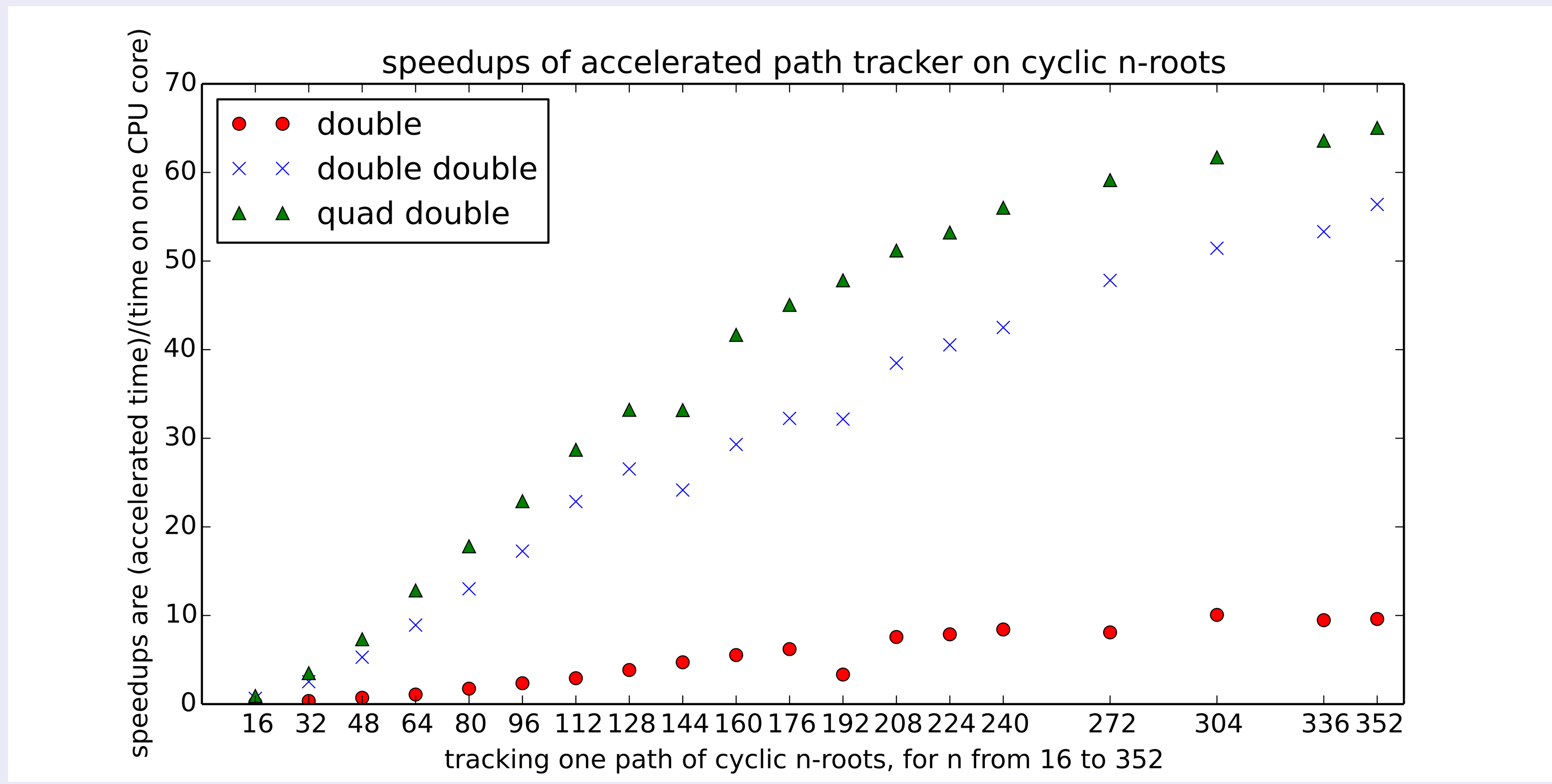
• message passing (with Yusong Wang, Yun Guan, Anton Leykin)
By design, the main program is written in C, responsible for the job scheduling,
with the aid of MPI. The jobs execute Ada procedures.

• multicore shared memory programming (with Genady Yoffe)
The goal of this project is to offset the cost of double double and quad double
arithmetic with multithreaded code.
`phc -b -t4` runs path trackers in the blackbox solver with 4 threads, using the
tasking mechanism provided by Ada (multitasking can be called in phcpy).

• acceleration with graphics processors (with Xiangcheng Yu).
The code is a mix of Ada, C, and C++ CUDA.

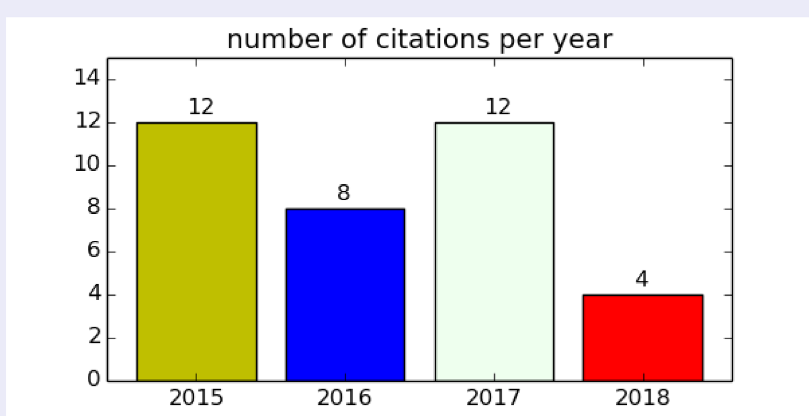
accelerating polynomial homotopy continuation

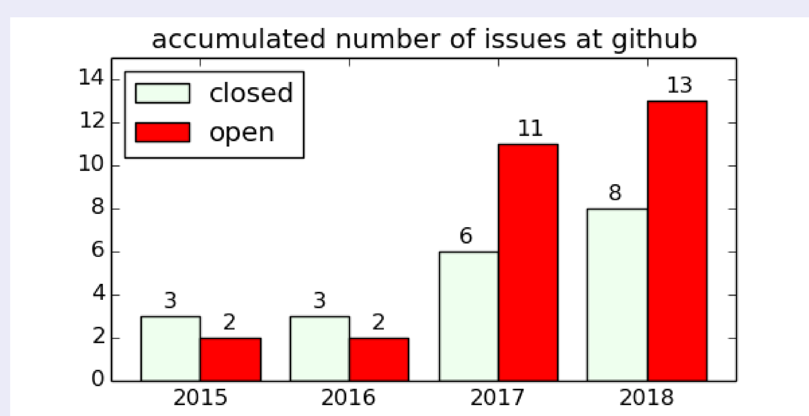
Results on tracking one path of the cyclic n-roots benchmark problem,
accelerated on the NVIDIA K20C, using GQD [Lu, He, and Luo 2010]:



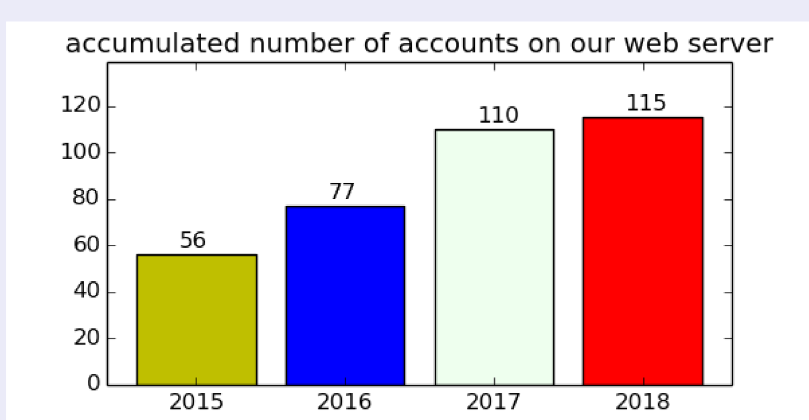
Double digit speedups allow to compensate for the overhead caused by complex
double double and quad double arithmetic.
Joint with Xiangcheng Yu (HPCC 2015, PASCO 2015).

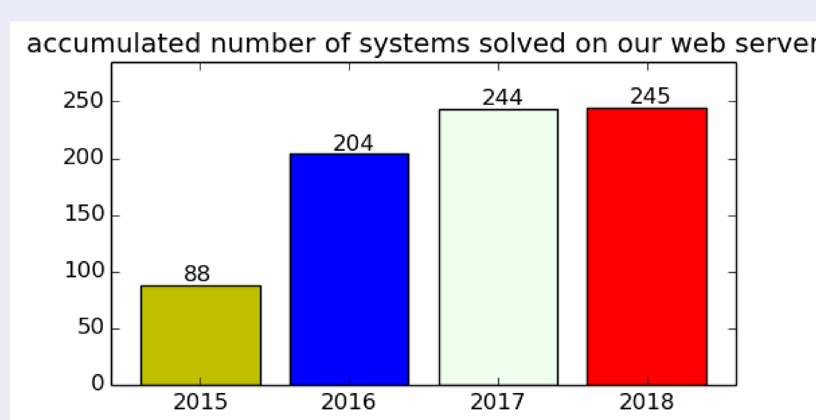
metrics: citations and issues





metrics: accounts and systems solved





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