Polynomial Homotopy Continuation on GPUs (software presentation)

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Polynomial Homotopy Continuation on GPUs

$f(\mathbf{x}) = \mathbf{0}$ is a polynomial system we want to solve,
$g(\mathbf{x}) = \mathbf{0}$ is a start system (g is similar to f) with known solutions.

A homotopy $h(\mathbf{x}, t) = (1 - t)g(\mathbf{x}) + t\, f(\mathbf{x}) = \mathbf{0}$, $t \in [0, 1]$,
to solve $f(\mathbf{x}) = \mathbf{0}$ defines solution paths $\mathbf{x}(t)$: $h(\mathbf{x}(t), t) \equiv \mathbf{0}$.

Numerical continuation methods track the paths $\mathbf{x}(t)$, from $t = 0$ to 1.

Problem: Graphics Processing Units (GPUs) provide better performance than Central Processing Units (CPUs) but operate along the Single Instruction Multiple Thread (SIMT) execution model.

Solution: the reverse mode of Algorithmic Differentiation (AD) provides fine grained parallelism to evaluate and differentiate all polynomials in the homotopy, as needed for Newton's method.

Acceleration compensates for the cost of double double and quad double arithmetic by the QD library (CPU) and the GQD library (GPU).

PHC on Graphics Processing Units

Long term goal: bring High Performance Computing into PHC.

Massively parallel path tracking (applying AD):

- one path: #monomials $\geq 10,000$,
- many paths: #paths $\geq 10,000$.

We need to keep 10,000 threads occupied.

Double double and quad double arithmetic (using QD):

- ***memory bound*** for double and (real) double double arithmetic,
- ***compute bound*** for complex double doubles and quad doubles.

Double digit speedups $\Rightarrow$ double the precision, compute twice as fast.

Free and open source software:

- released under GNU GPL, latest version on github,
- developed in C++ with NVIDIA CUDA, versions 5.0 to 6.5,
- on the NVIDIA K20C, its host provides our cloud service.

High Level Parallel Path Tracking with `phcpy`

`phcpy` is the Python scripting interface to PHCpack.

The path trackers have the number of tasks as extra option:

```
>>> from phcpy.trackers \
import standard_double_trackers as track
>>> endsols = track(target, start, sols, tasks = 4)
```

where

- `target` contains the polynomials of the target system $f(\mathbf{x}) = \mathbf{0}$,
- `start` contains the polynomials of the start system $g(\mathbf{x}) = \mathbf{0}$,
- `sols` contains the solutions of the system $g(\mathbf{x}) = \mathbf{0}$,

and `tasks = 4` indicates that four threads will be used.

Another interactive feature: give the next solution on a path, in a precision as set by condition number estimates.

the QD and GQD libraries

Software we used for double double and quad double arithmetic:

- Y. Hida, X. S. Li, and D. H. Bailey. **Algorithms for quad-double precision floating point arithmetic.** In *15th IEEE Symposium on Computer Arithmetic (Arith-15 2001)*. `http://crd.lbl.gov/~dhbailey/mpdist`
- M. Lu, B. He, and Q. Luo. **Supporting extended precision on graphics processors.** In *Proceedings of the Sixth International Workshop on Data Management on New Hardware (DaMoN 2010)*. `http://code.google.com/p/gpuprec/`

our papers

- **Solving polynomial systems in the cloud with polynomial homotopy continuation.** (with N. Bliss, J. Sommars, and X. Yu). *Proceedings of CASC 2015.* `arXiv:1506.02618`
- **Evaluating polynomials in several variables and their derivatives on a GPU computing processor.** (with G. Yoffe). *Proceedings of PDSEC 2012.* `arXiv:1201.0499`
- **Orthogonalization on a general purpose graphics processing unit with double double and quad double arithmetic.** (with G. Yoffe). *Proceedings of PDSEC 2013.* `arXiv:1210.0800`
- **Acceleration of Newton's method for large systems of polynomial equations in double double and quad double arithmetic.** (with X. Yu). *Proceedings of HPCC 2014.* `arXiv:1402.2626`
- **Accelerating polynomial homotopy continuation on a graphics processing unit with double double and quad double arithmetic.** (with X. Yu). *Proceedings of PASCO 2015.* `arXiv:1501.06625`
- **Tracking many solution paths of a polynomial homotopy on a graphics processing unit with double double and quad double arithmetic.** (with X. Yu). *Proceedings of HPCC 2015.* `arXiv:1505.00383`