Solving Polynomial Systems with *phcpy*

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PHCpack is a package for **polynomial homotopy continuation** to solve polynomial systems. *ACM Transactions on Mathematical Software* has archived version 1.0 as Algorithm 795, vol. 25, no. 2, pages 251--276, 1999.

Two **blackbox solvers** in phcpy can compute:
1) the isolated solutions of a polynomial system.
2) a numerical irreducible decomposition, i.e., all *solution sets* of the system.

phcpy exposes the functionality of PHCpack to Python scripts. Computationally intensive algorithms are executed efficiently by the compiled code.

DISCUSSION

Usage of *phcpy* has been reported in research literature:

Symbolic computation – # embeddings of minimally rigid graphs (Bartzos, Emiris, Legersky, and Tsigaridas, 2018)

Pure math – Roots of Alexander polynomials (Culler and Dunfield, 2018)

Chemical engineering – Critical points of equilibrium problems (Sidky, Whitmer, and Mehta, 2016)

Numerical algebraic geometry is currently used in other fields whose models rely on nonlinear polynomial systems:

Rigid-body mechanisms – Algebraic kinematics for synthesis and control of motion, e.g. in robotics or animation. (Wampler and Sommese, 2011)

Systems biology – Model selection via analysis of steady states, e.g. in pathway analysis or disease modeling. (Gross, Davis, Ho, Bates, and Harrington, 2016)

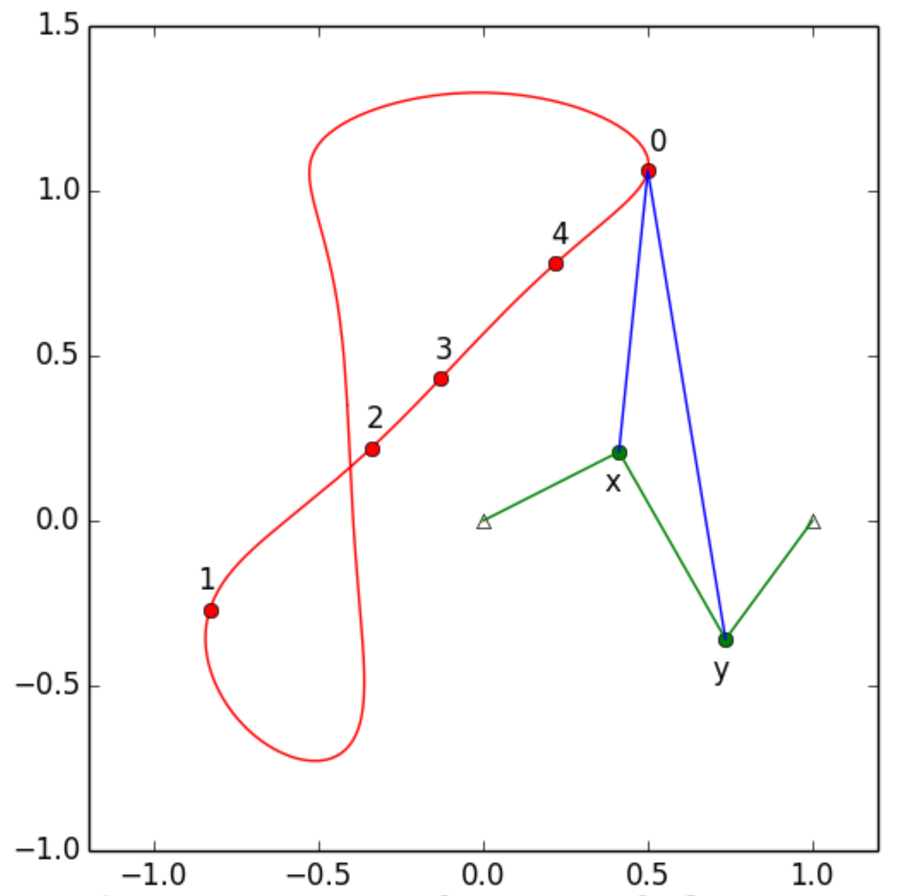
We can numerically solve polynomial systems in *phcpy*, using fast and reliable homotopy continuation methods.

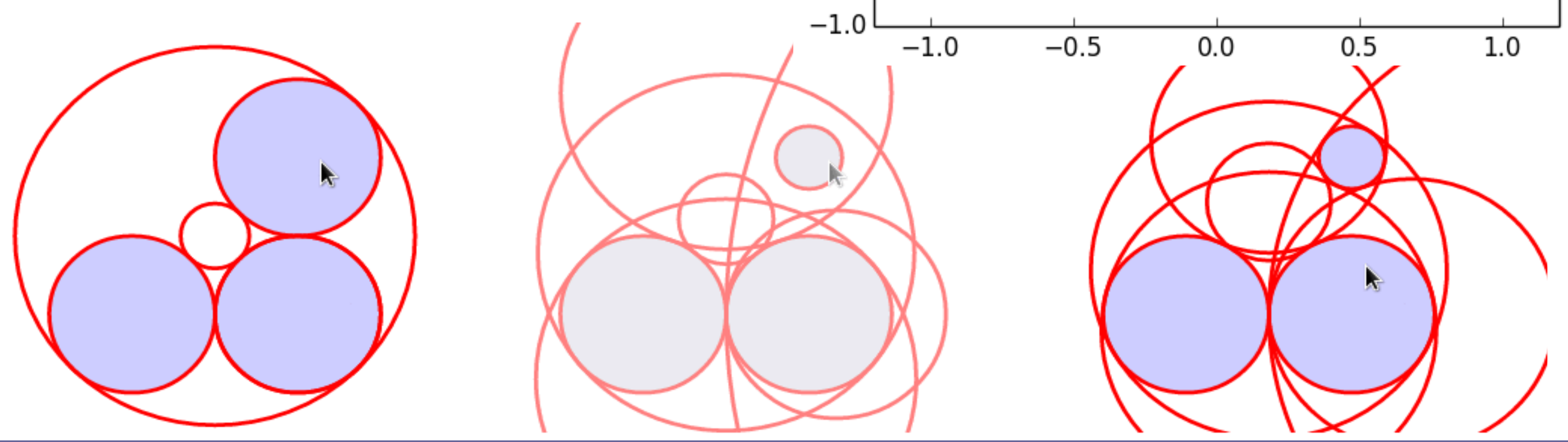
TUTORIALS math.uic.edu/~jan/phcpy_doc_html/usecases.html

(right) Given 5 precision points, design a 4-bar mechanism.

(below) Compute all circles that touch three given circles.

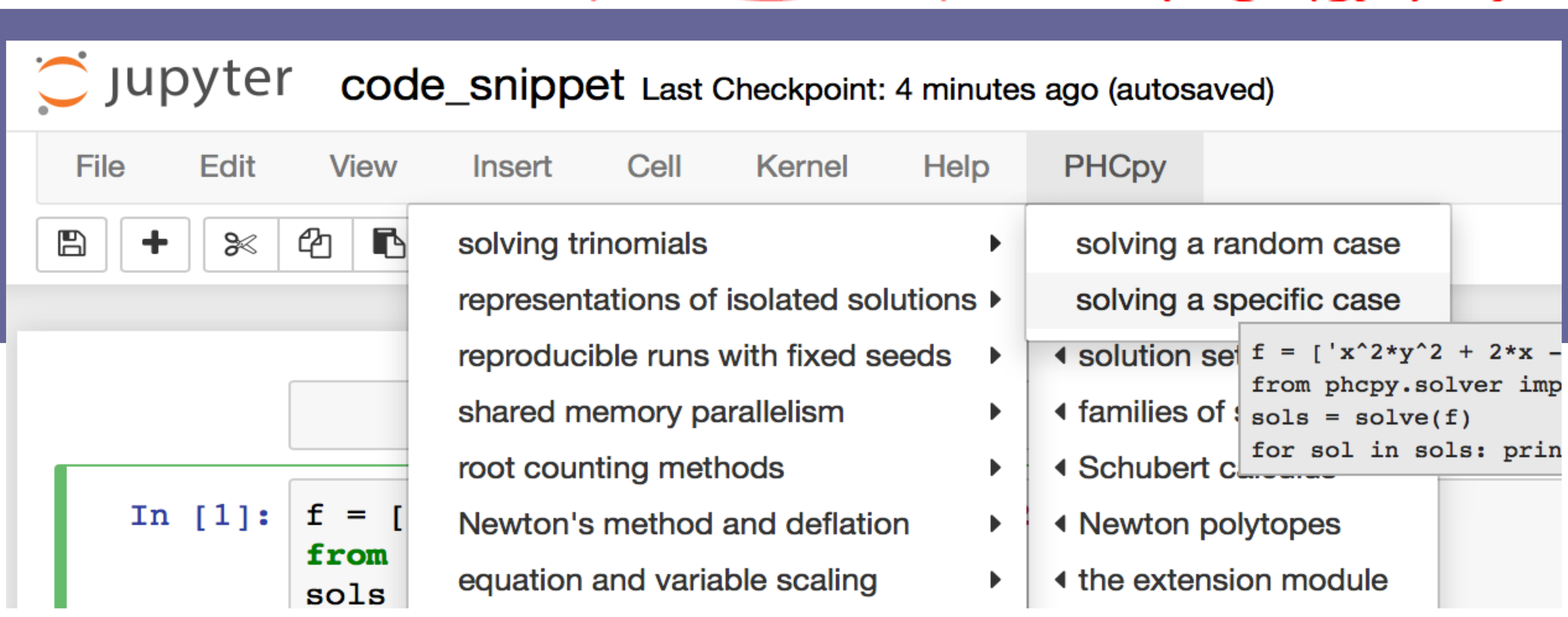
The *phcpy* documentation includes these tutorials. Further use cases are documented in our paper, conference.scipy.org/proceedings/scipy2019/pdfs/jan_verschelde.pdf





A web interface to phcpy is online at www.phcpack.org

, a notebook with Python and SageMath kernels, available for public use with a free account.



INTERACTIVE PARALLELISM

```
def qualityup(nbtasks=0, precflag='d'):
    from phcpy.families import cyclic
    from phcpy.solver import solve
    from time import perf_counter
    c7 = cyclic(7)
    tstart = perf_counter()
    s = solve(c7, verbose=False, tasks=nbtasks,\
        precision=precflag, checkin=False)
    return perf_counter() - tstart
```

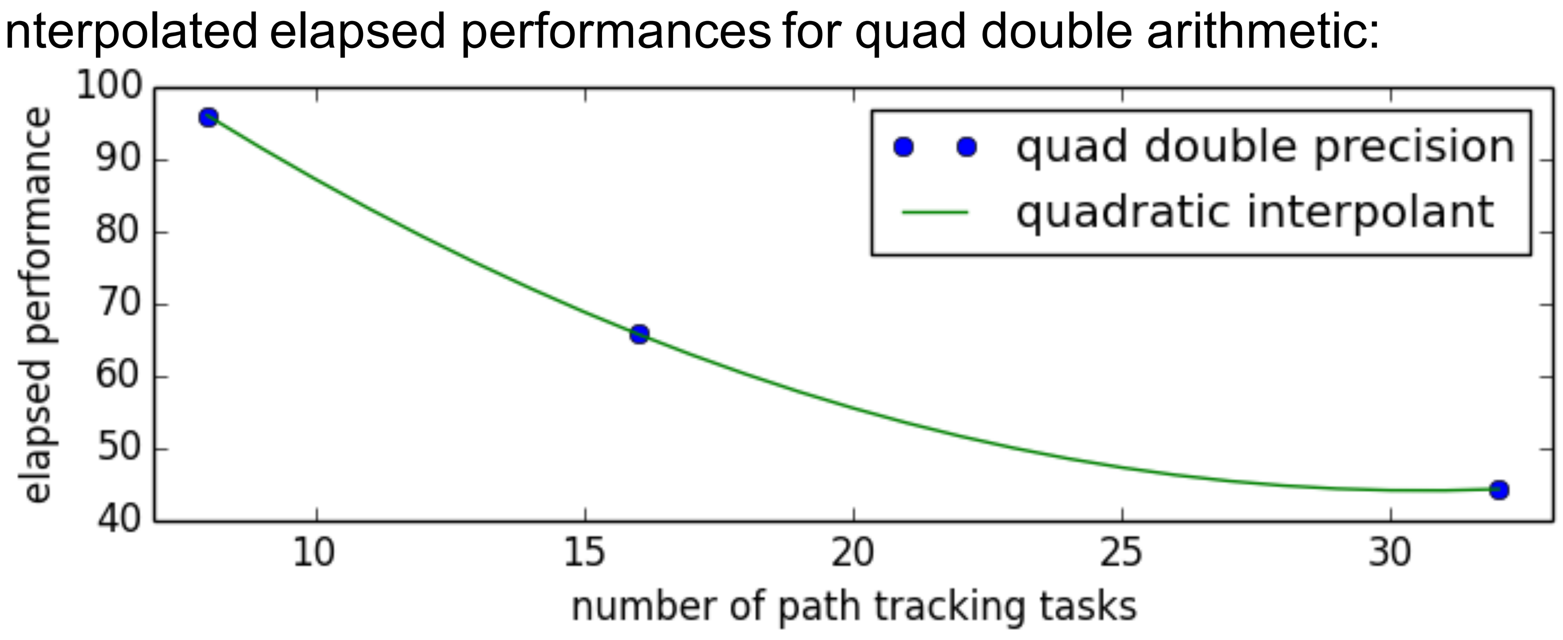
The blackbox solver's elapsed performance in double, double double, and quad double precision:

precision	d	dd	qd
Elapsed performance	5.45	**42.41**	604.91
Overhead factor	1.00	7.41	110.99

tasks	8	16	32
dd	7.56	5.07	3.88
qd	96.08	65.82	**44.35**

The overhead of extra precision is compensated by multithreading.

Interpolated elapsed performances for quad double arithmetic:



SOLUTION SETS

A numerical irreducible decomposition includes **representations** for all positive dimensional solution sets.

Consider two equations defining the *twisted cubic*:

pols = ['x*y - z;', 'x^2 – y;']

(1) A *witness set* provides generic points:

```
from phcpy.sets import embed
from phcpy.solver import solve
embp = embed(3, 1, pols)
sols = solve(embp, verbose=False)
print('#generic points :', len(sols))
```

Three points lie at the intersection of this cubic with a random plane.

(2) A *series expansion* develops from some point(s) in a coordinate hyperplane. For the twisted cubic, the series is exact after the first term.

```
from phcpy.maps import solve_binomials
maps = solve_binomials(3, pols, puretopdim=True)
for sol in maps: print(sol)
```

The solution gives (t,t^2,t^3), the parametric representation of the twisted cubic.

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