Outline

Simulation
- Monte Carlo methods
- random numbers

Repeat Until
- binary expansion
- break statement

stats in Maple

Summary + Assignments

MCS 260 Lecture 12
Introduction to Computer Science
Jan Verschelde, 24 September 2007

Simulation

Monte Carlo methods

- In a mathematical model with uncertainties, events occur with assigned probabilities.
- Simulation consists in the repeated drawing of samples according to a probability distribution. We count the number of successful samples.
- The Law of Large Numbers states that the arithmetic average of the observed successes converges to the expected value or mean of the experiment, as the number of experiments increases.
- Monte Carlo methods are listed among the Top Ten Algorithms of the 20th century.

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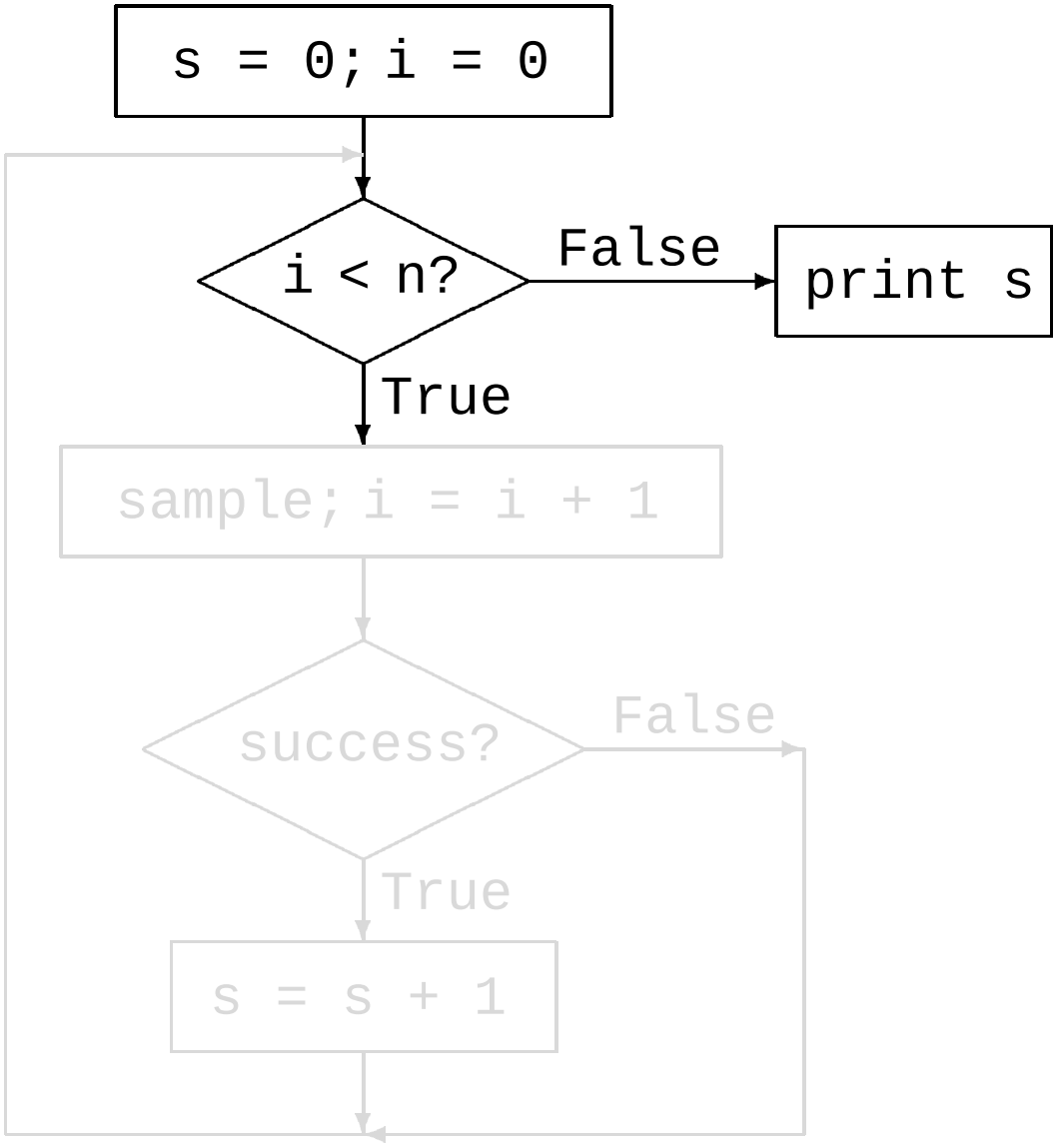
Flowchart for general Simulations



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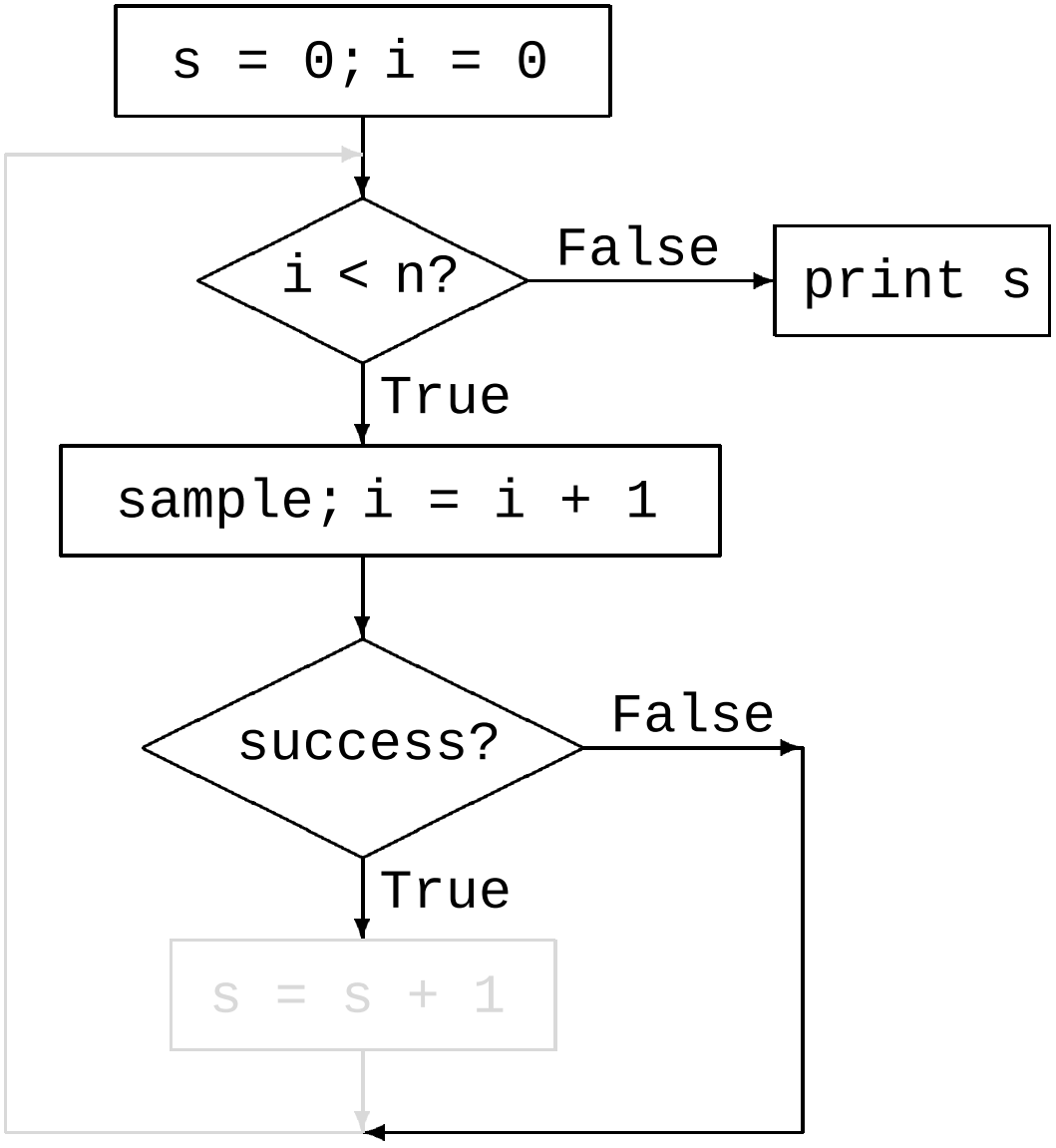

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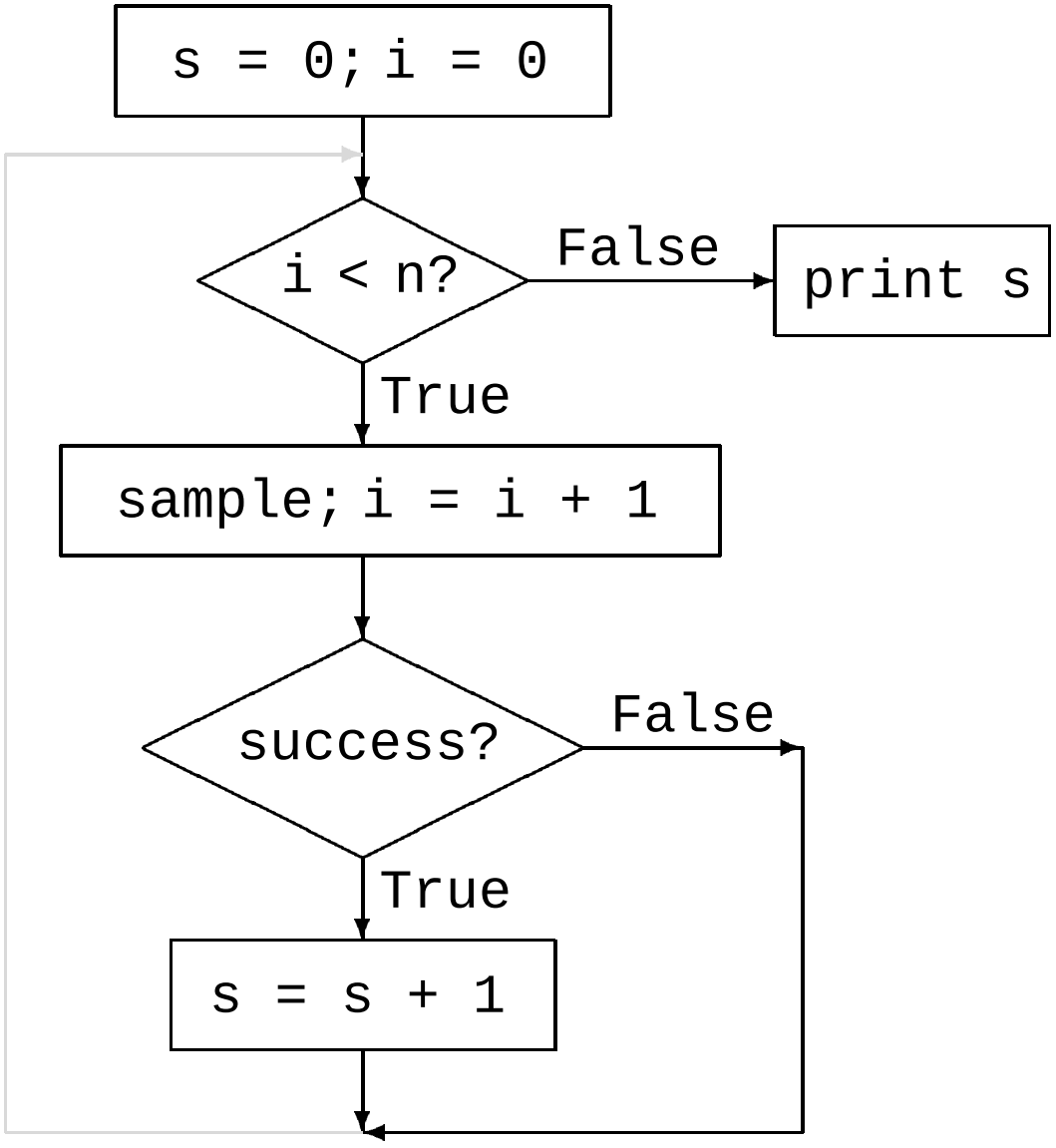

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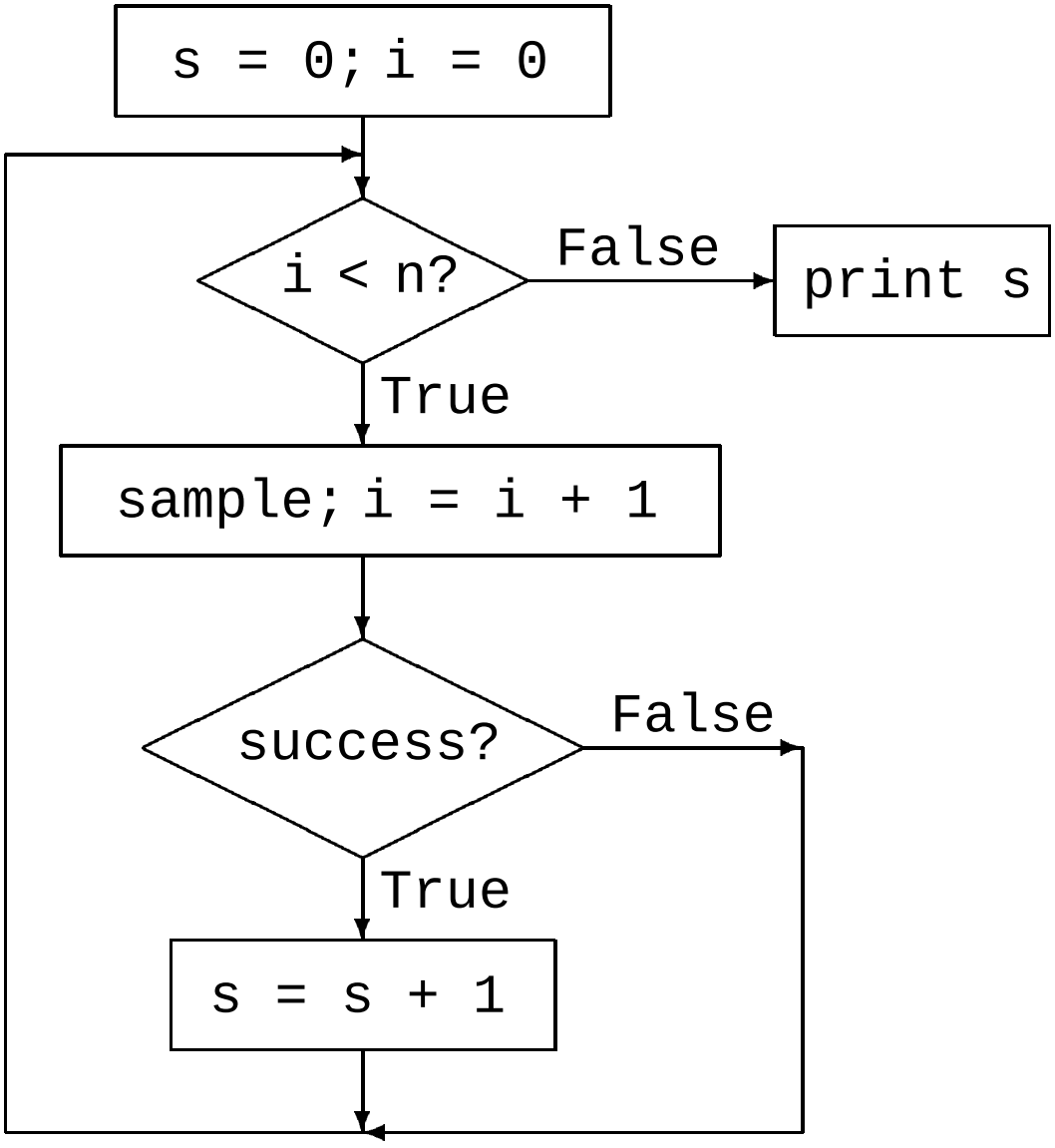

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Random Numbers

as available in Python

Random number generators are in the module `random`.

Three things we need to know:

1. `import random` loads the module into a session. Afterwards, `help(random)` shows a description of the definitions and functions offered by the module.

2. `random.seed()`
   Giving a fixed number as argument results in the same sequence of random numbers.

3. `r = random.uniform(a,b)`
   `r` is a randomly generated number, drawn from a uniform distribution over the interval `[a,b)`.

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Using Random Numbers

a sample program

A sample program randuse.py:

```
# L-12 MCS 260 Mon 24 Sep 2007  random numbers
#
import random          # use module random
random.seed(21342342) # get same sequence
print 'uniformly distributed random numbers'
a = input('give lower bound : ')
b = input('give upper bound : ')
r = random.uniform(a,b) # generate a number
print 'a random number in [%.2f,%.2f] : %.15f' \
      % (a,b,r)
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Estimating Areas and Volumes

high dimensional integrals

- Expected values are expressed as integrals. When many parameters are involved, the integration is high dimensional and only estimation is possible.
- The area of the unit disk is π.

Generate random uniformly distributed points with coordinates $(x, y) \in [-1, +1] \times [-1, +1]$.

We count a success when $x^2 + y^2 \leq 1$.

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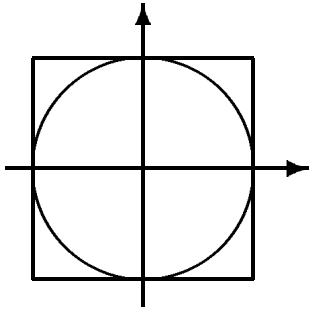

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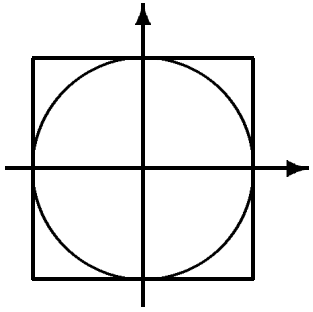

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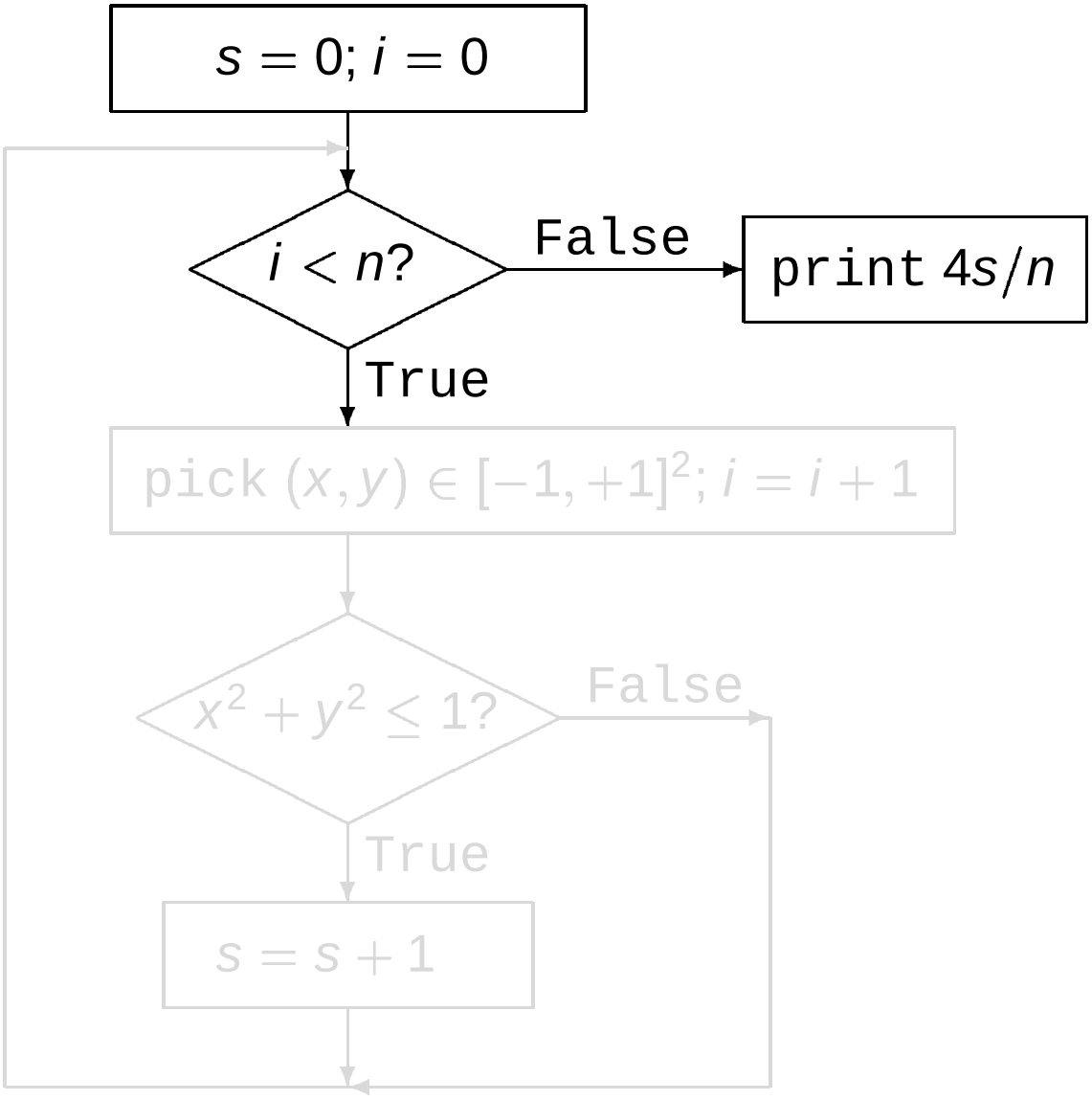
Flowchart for Estimating π



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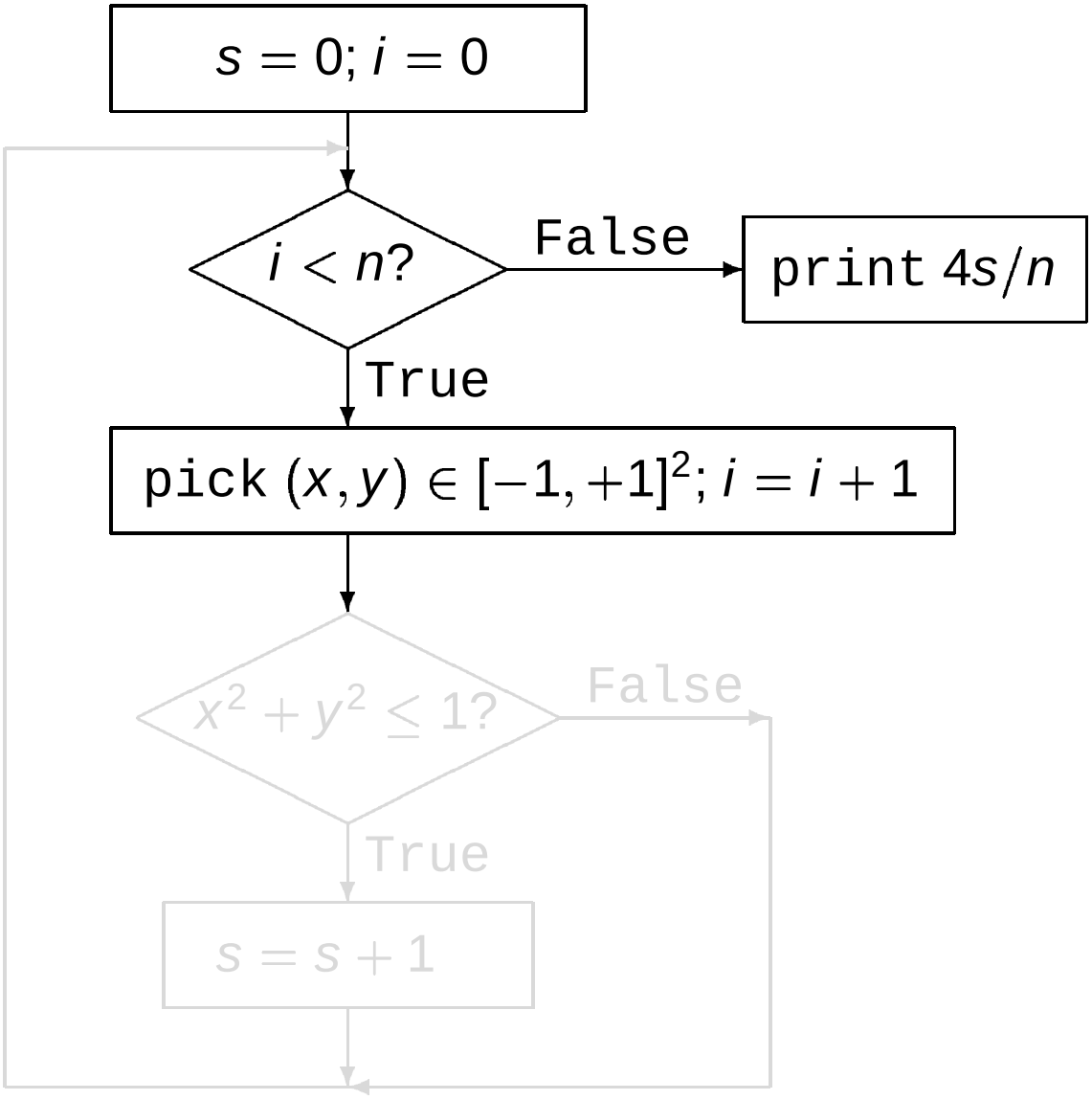

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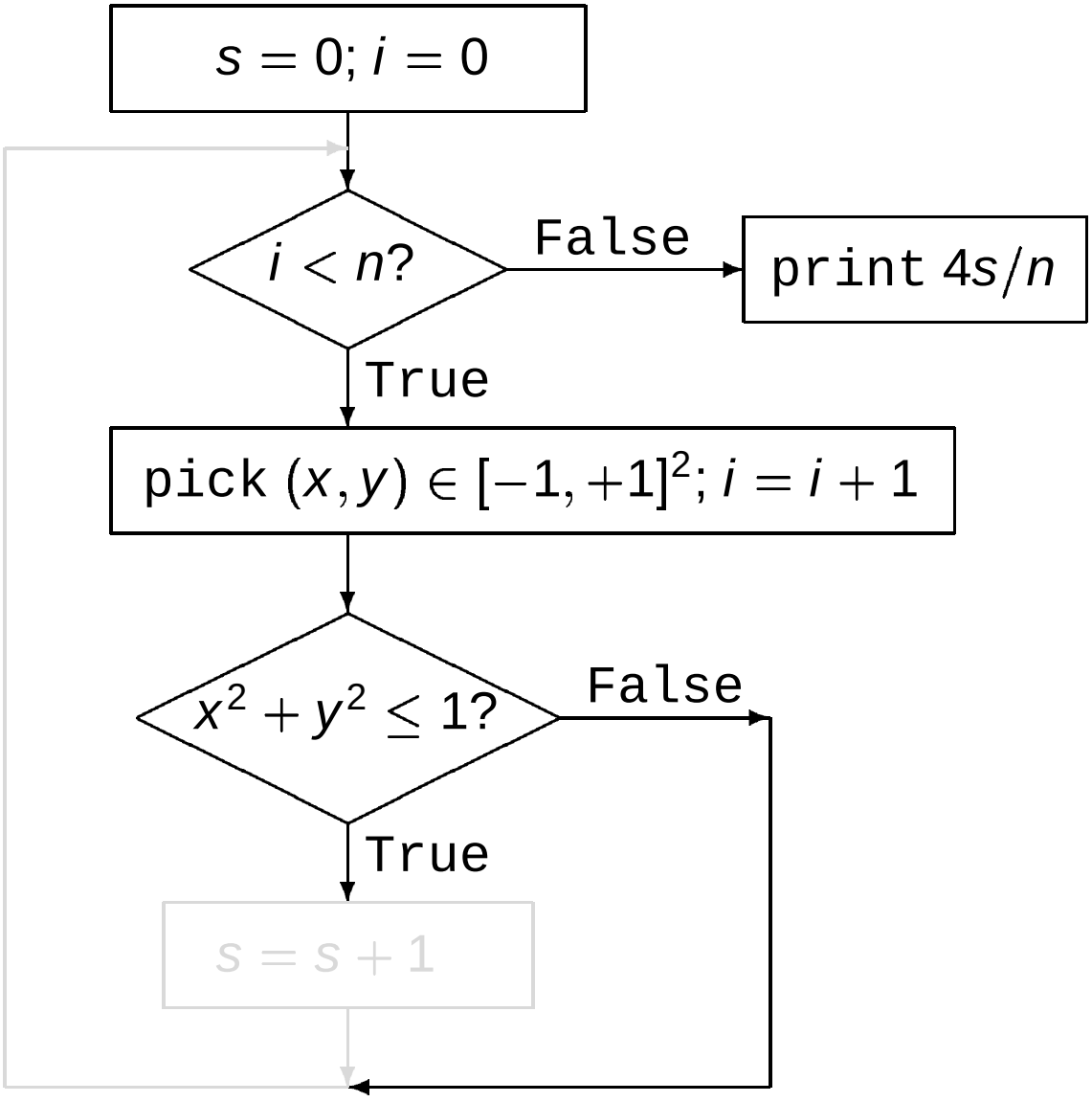

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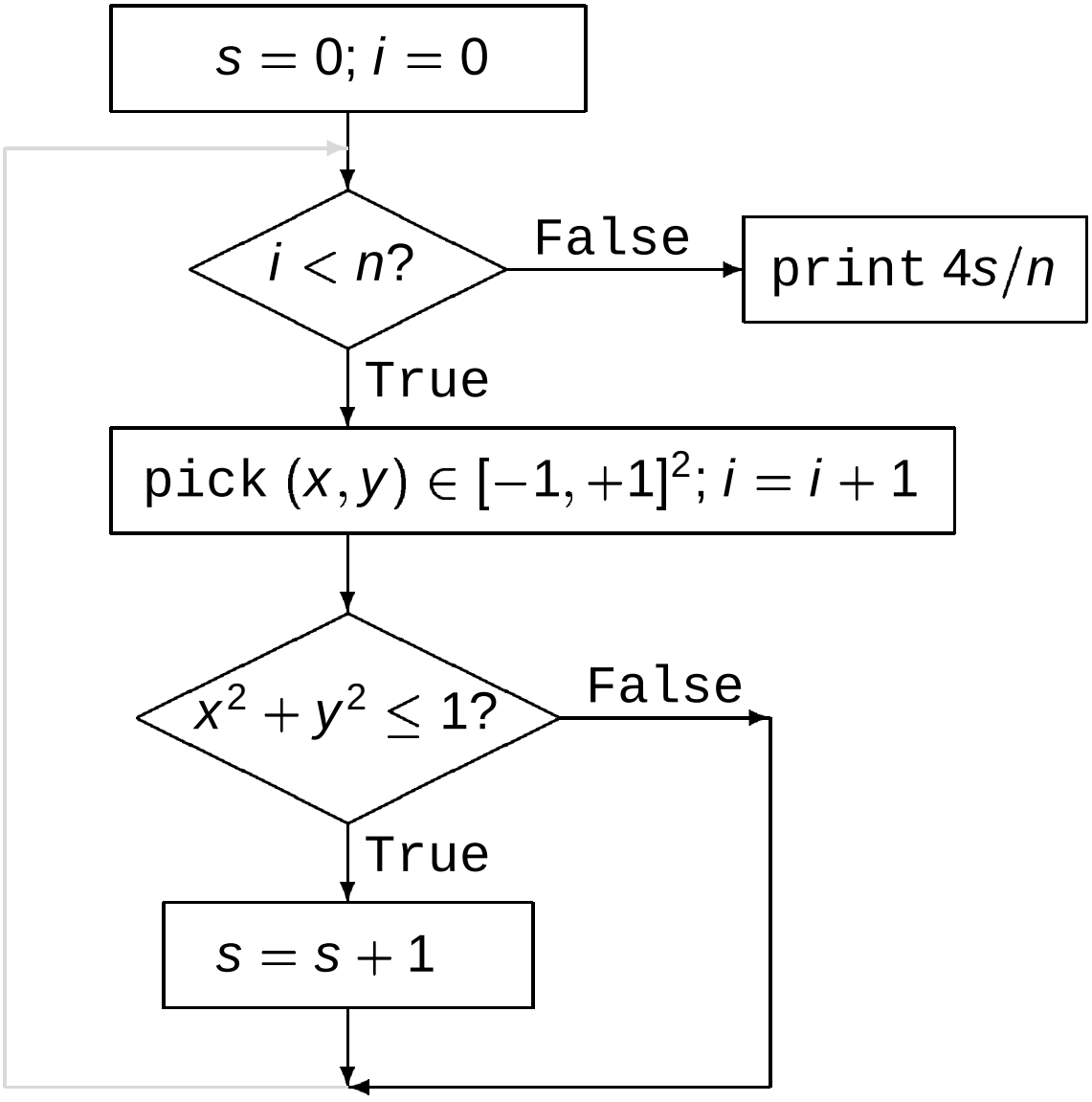

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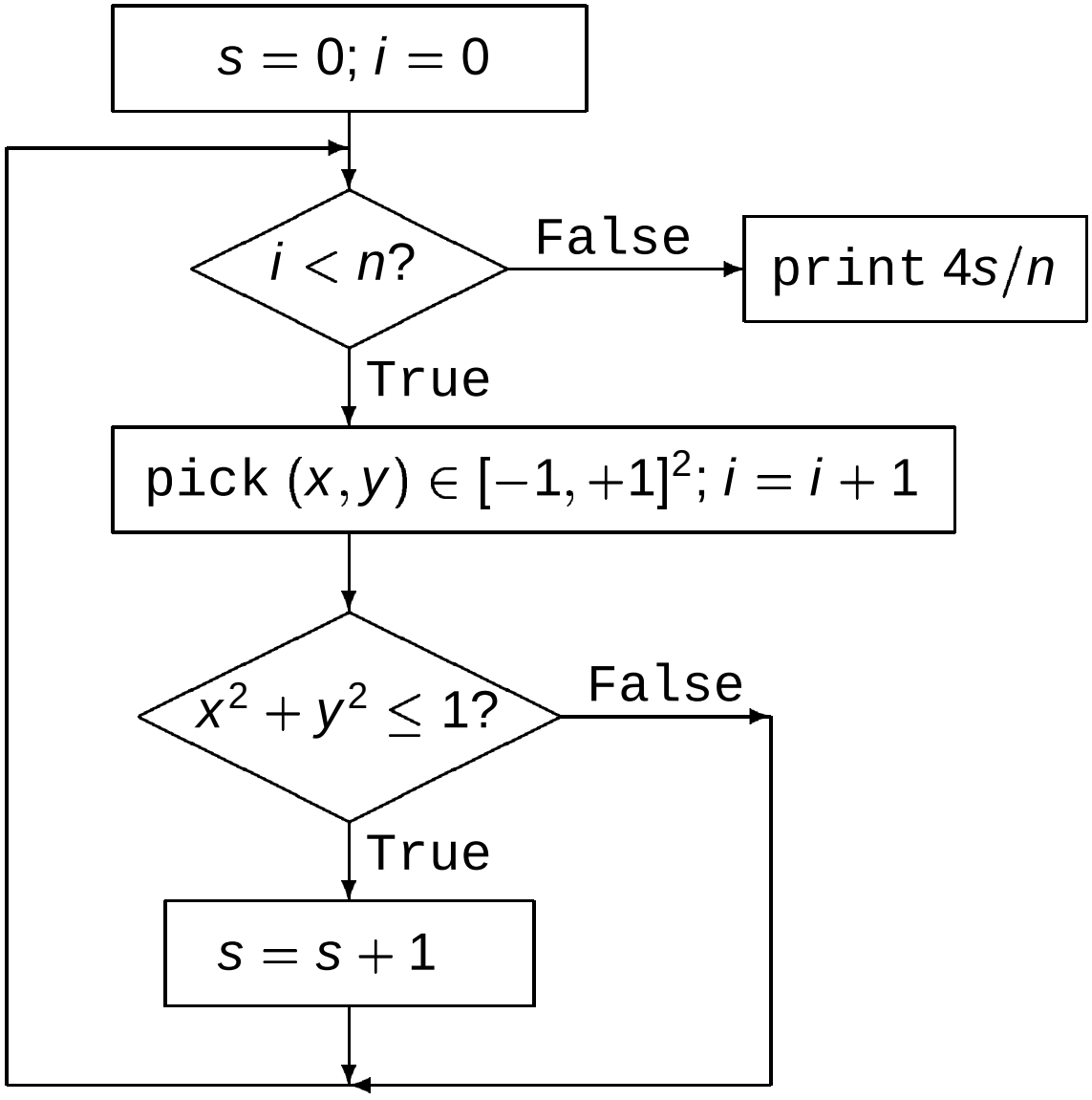

Estimating π

The program mc4pi.py:

```
# L-12 MCS 260 Mon 24 Sep 2007  Monte Carlo
#
import random
print 'Monte Carlo simulation for Pi'
n = input('Give number of runs : ')
s = 0
for i in range(0,n):
   x = random.uniform(-1,1)
   y = random.uniform(-1,1)
   if x**2 + y**2 <= 1: s += 1
print 'After %d runs : %f' % (n,4.0*s/n)
```

Why multiply by 4? 4 is the area of $[-1,+1]^2$.

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Binary Expansions

repeat until loops

The bits of a number are the remainders of division by 2.

divmod() is an intrinsic operation:

```
>>> divmod(9,2)
(4, 1)
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Use as `(n,r) = divmod(n,2)`
to obtain remainder n%2 in `r` and to replace n by n/2.

Pseudo code to compute the binary expansion:

```
n = input()
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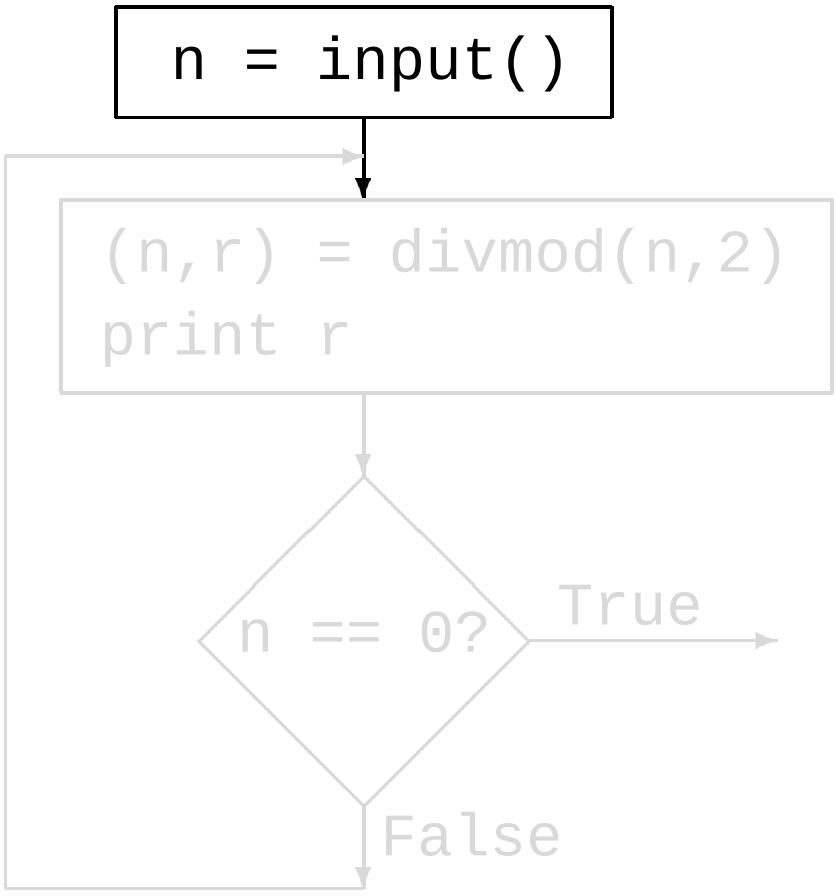

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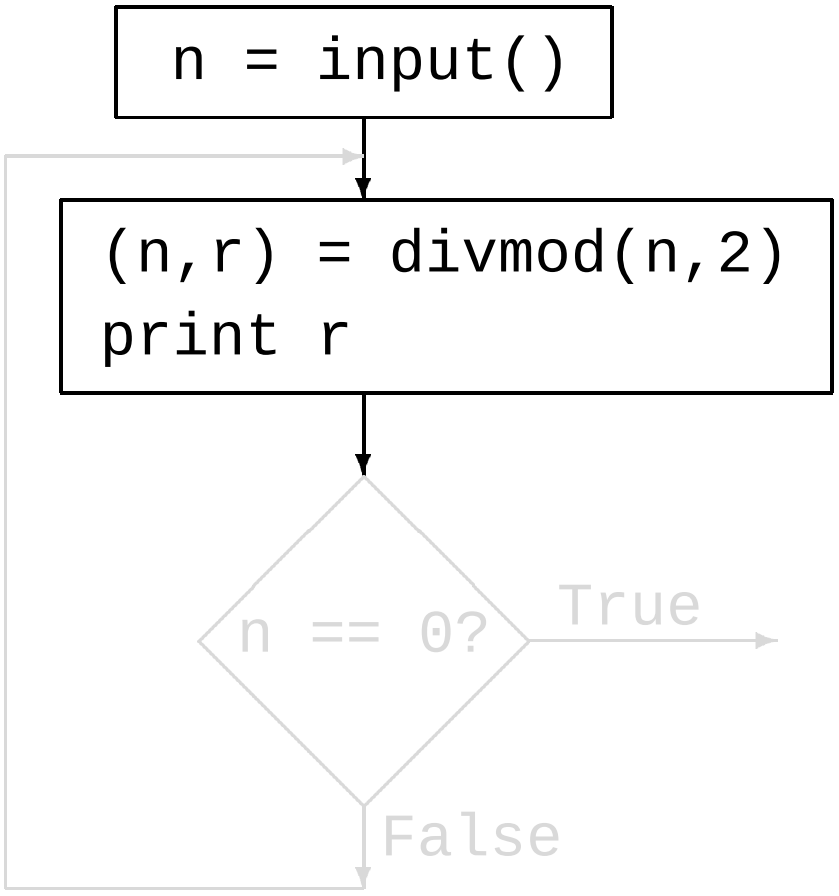

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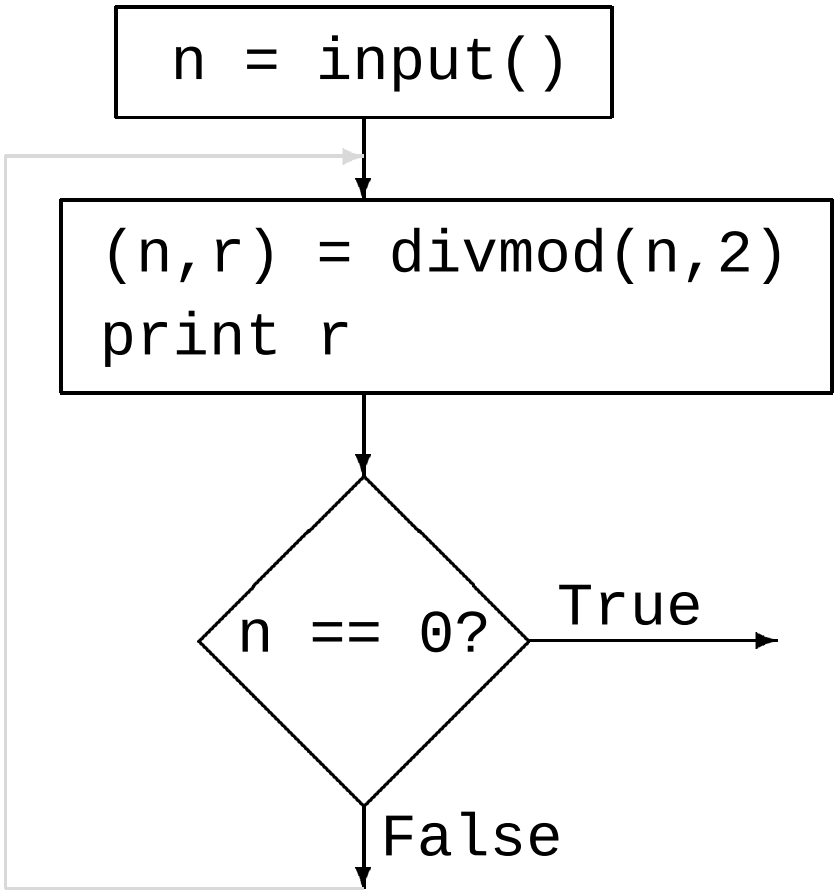

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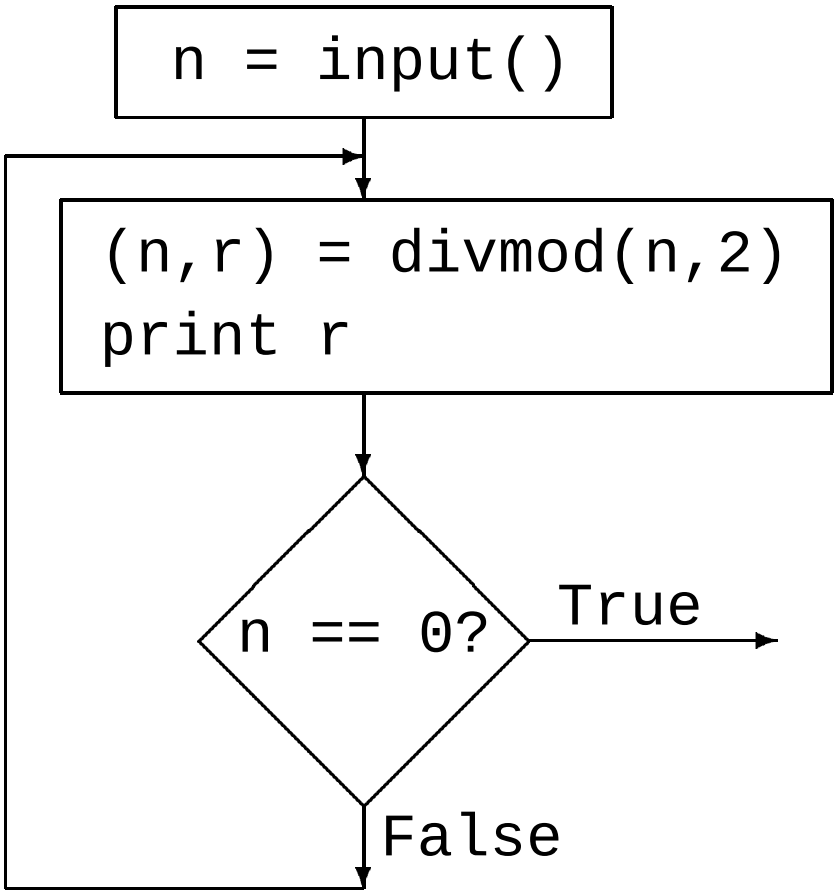

A first Python solution

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# This first version prints the bits
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#
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n = input('Give a number : ')
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print r
while n > 0:
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The break Statement

repeat until as while true break

To exit a loop inside the body of a loop, the statement `break` occurs usually within an `if` statement.

```
repeat
   < body of loop >
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is realized in Python as

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while True:
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The `while True` starts an infinite loop, terminated when `< condition >` becomes True.

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Binary Expansions with break

a better solution

The program below avoids the duplication of code:

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# L-12 MCS 260 Mon 24 Sep 2007  binary expansion
#
# Use of break for repeat until.
#
print 'computing the binary expansion'
n = input('Give a number : ')
while True:
   (n,r) = divmod(n,2)
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Exercise: how to print bits in correct order?

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Two Loops, Two Breaks

scope of a break

As long as the number typed in by the user is nonnegative, the loop continues.

```
# L-12 MCS 260 Mon 24 Sep 2007  binary expansion
#
# A break only effects one loop.
#
print 'computing the binary expansion'
while True:
   n = input('Give a number (< 0 to exit) : ')
   if n < 0: break
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A break only effects the one loop it is in.

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Random Numbers in Maple

Maple offers random numbers generators for various probability distributions:

```
> with(stats[random]);
```

The `uniform` returns uniformly distributed numbers.

Generating 5 random numbers in $[-1, +2]$:

```
> uniform[-1,+2](5);
```

or without loading the package `stats[random]`:

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> stats[random,uniform[-1,+2]](5);
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Summary + Assignments

We covered in *Making Use of Python*:

- towards the end of chapter 4;
- some of the topics in chapter 6.

Assignments:

1. Use a stack to store the bits in the binary expansion to print the bits *after* the loop in the correct order.
2. Given a list of numbers between 0 and 100, define the algorithm to assign a letter grade to each number: ≥ 90: A, $\in [80, 89]$: B, $\in [70, 79]$: C, etc. Report at the end how many As, Bs, Cs, etc. Write the algorithm in words and draw a flowchart.
3. Implement exercise 2 in Python.
4. Write a Python program that generates n numbers uniformly distributed in $[0, 1]$ and counts how many numbers are < 0.5.