# Outline



MCS 260 Lecture 8
Introduction to Computer Science
Jan Verschelde, 29 January 2016

# Boolean Algebra, Flowcharts
# Conditional Expressions

# Boolean Algebra

## computing with logical expressions

Boolean algebra is the calculation with `True` and `False` (often having values 1 and 0). The operators are `and`, `or`, and `not`. Truth tables define the outcome for all values:

| x | y | x and y |
|---|---|---|
| False | False | False |
| False | True | False |
| True | False | False |
| True | True | True |

| x | y | x or y |
|---|---|---|
| False | False | False |
| False | True | True |
| True | False | True |
| True | True | True |

# Evaluation Laws

## law and order in the Boolean algebra

When `not`, `and`, `or` occur in an expression,
`not` is first evaluated, before `and`, and finally `or`.

De Morgan's laws for simplifying expressions:

- `not (( not x ) or ( not y )) = x and y`
  Negating not being alive or not being well
  means being alive and being well.
- `not (( not x ) and ( not y )) = x or y`
  Negating not going to school and not going to work
  means going to school or going to work.

We prove these laws by truth tables.
Applications of truth tables:

- Realization of electronic circuits.
- Simplification of conditional statements in programs.

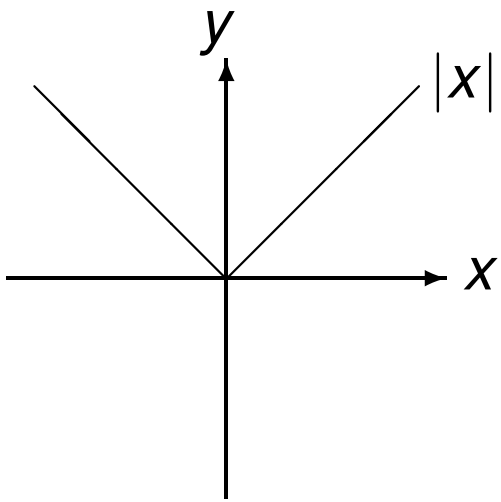
# Boolean Algebra, Flowcharts
# Conditional Expressions

# The Absolute Value – an example of an `if` statement

The function `abs` is available in Python:

```
>>> abs(-3.5)
3.5
>>> abs(3.5)
3.5
```

The mathematical definition of `abs`($x$) as $y = |x|$:




$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

# Pseudocode

## to formally describe algorithms

To develop and define an algorithm, we use *pseudocode*. Pseudocode is not real code, but to the reader it has the same properties as a formal language.

Example: print the absolute value of a number.
The number is given by the user.

In words, we could describe the program as:

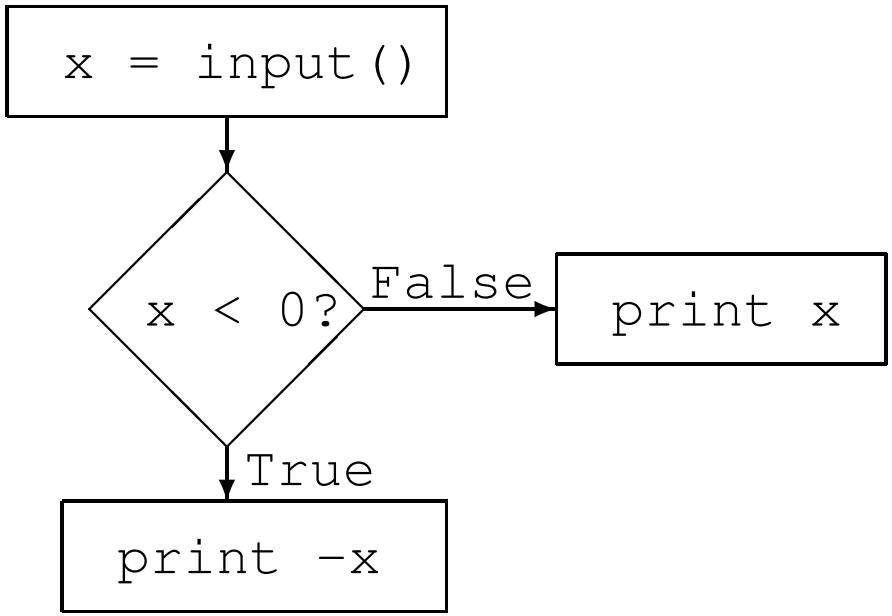
ask the user for a number;
`if` the number is less than zero,
`then` print – before the number;
`else` print the number.

Mix of formal `if`, `then`, and `else` with English.

# Flowcharts

## pictures of algorithms

Printing the absolute value of a number:




Flowcharts schematically represent the logical flow.

# Boolean Algebra, Flowcharts
# Conditional Expressions

1. Boolean Algebra
   - logical expressions
   - pseudocode and flowcharts
2. Conditional Constructs
   - conditional operators
   - if, else, elif
3. Logic in Sage
   - computing truth tables with Sage
4. Summary + Assignments

# Comparison Operators

The outcome of a comparison is `True` or `False`:

```
>>> 1 < 7
True
>>> 1 >= 7
False
```

The comparison operators:

| | | | |
|---|---|---|---|
| `==` | : | `x == y` | is equal? |
| `!=` | : | `x != y` | not equal? |
| `<` | : | `x < y` | less than? |
| `>` | : | `x > y` | greater than? |
| `<=` | : | `x <= y` | less or equal? |
| `>=` | : | `x >= y` | greater or equal? |

# The `is` Operator – equal values but different objects

Testing whether composite objects are equal:

```
>>> L = [2,3]; K = [2,3]
>>> L == K
True
>>> L is K
False
```

`L` and `K` refer to

- lists which contain the **same values**,
- lists stored as **different objects**.

```
>>> M = L; M is L
True
>>> M == L
True
```

# Boolean Operators

Combining results of logical expressions:

```
>>> x = 3
>>> (x > 0) and (x < 10)
True
>>> (x < 0) or (x < 5)
True
>>> not (x < 0)
True
```

The brackets are not needed.

| | | |
|---|---|---|
| `and` | `x and y` | both True? |
| `or` | `x or y` | is one True? |
| `not` | `not x` | is False? |

# Printing Booleans – as numbers or strings

Type `bool` is another elementary data type:

```
>>> type(True)
<type 'bool'>
```

Although `True` is 1 and `False` is 0:

```
>>> '%d' % True
'1'
>>> '%d' % False
'0'
```

Printing booleans as strings:

```
>>> str(True)
'True'
>>> '%s' % True
'True'
```

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# The if Statement – conditional execution of code

The syntax of the `if`:

```
if < condition >:
    < statements when condition is True >
```

All statements to be executed only if the condition is true must be preceded by the right intendations!

Suppose we want to print the '+' for positive numbers.

With an `if` we could do it as follows:

```
if x > 0:
    print '+'
    print x
```

```
if x > 0:
      print '+'
print x
```

Only the second one works correctly for all `x`.

# the if else statement

The syntax of the `if else`:

```
if < condition >:
    < statements when condition is True >
else:
    < statements when condition is False >
```

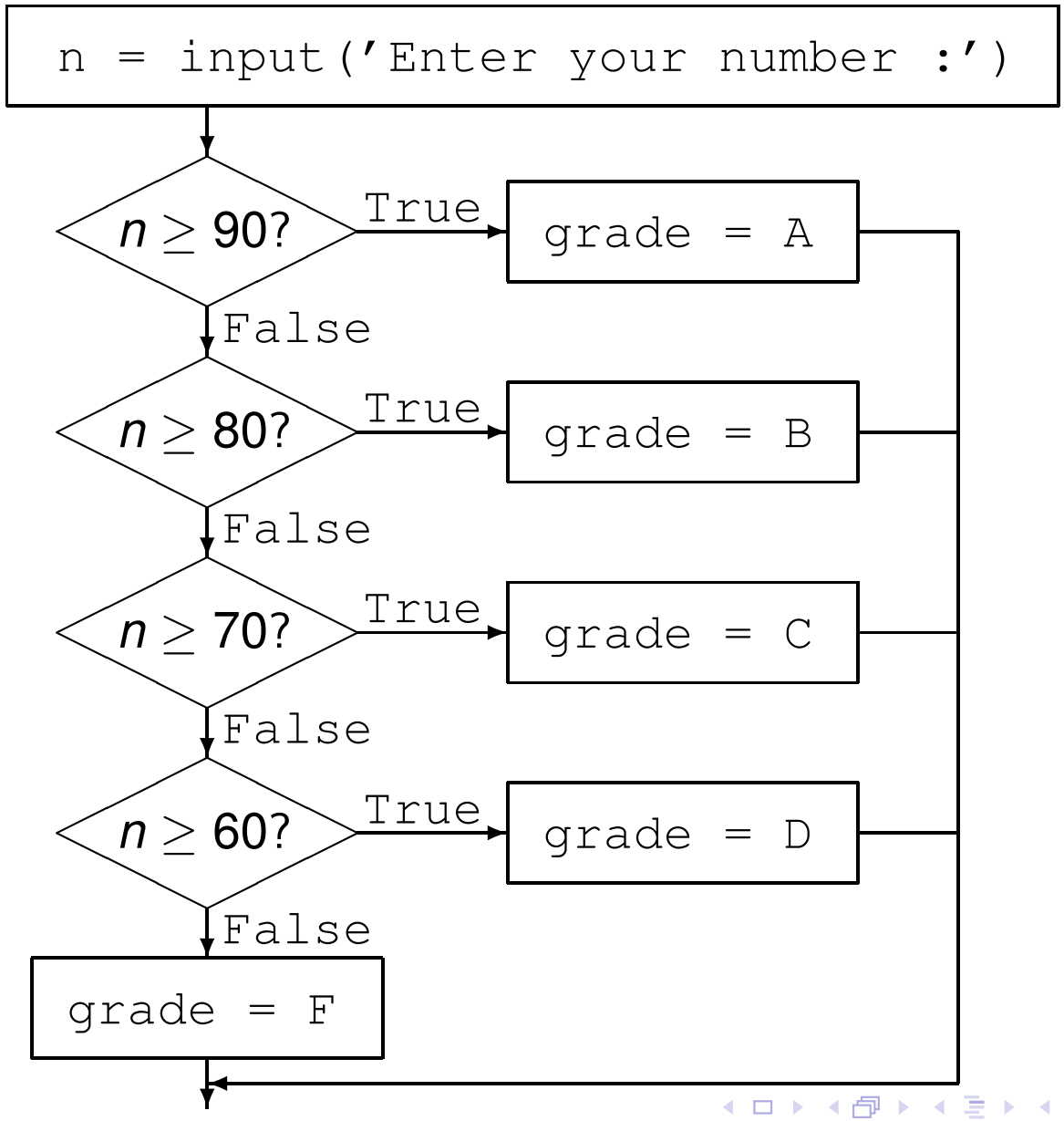
Printing the absolute value of a number:

```
if x < 0:
    print -x
else:
    print x
```

# Pass or Fail – illustration of an if else

```
"""
This program prompts the user to enter a number.
If the number entered is larger or equal than 80,
then the user is congratulated, else we are sorry.
"""
DATA = input('Enter your number : ')
NUMBER = int(DATA)
if NUMBER >= 80:
    print('Congratulations.  You passed!')
else:
    print('Sorry.  Please try again...')
```

# flowchart of grade scale



# if elif else to choose between multiple alternatives

The syntax of the `if elif else`:

```
if < condition 1 >:
    < statements when condition 1 is True >
elif < condition 2 >:
    < statements when condition 2 is True >
...
elif < condition n >:
    < statements when condition n is True >
else:
    < statements when everything is False >
```

The conditions are evaluated in the order as they appear.

# Showing the Grade – illustration of elif

The grade for a course is represented by a letter.
We compute the grade along a scale.

```
DATA = input('Enter your number : ')
NUMBER = int(DATA)
if NUMBER >= 90:
    GRADE = 'A'
elif NUMBER >= 80:
    GRADE = 'B'
elif NUMBER >= 70:
    GRADE = 'C'
elif NUMBER >= 60:
    GRADE = 'D'
else:
    GRADE = 'F'
print('Your grade is ' + GRADE + '.')
```

# Nested if else statements – follow up questions

Statements following `if` or else can again be conditional.
Nested if statements are good for dialogues with a user,
when the outcome cannot be anticipated:

```
ANS = input('happy ? (y/n) ')
if ANS == 'n':
    ANS = input('bored ? (y/n) ')
    if ANS == 'y':
        print('class is soon over')
    else:
        print('but it is Friday')
else:
    print('keep up the good work')
```

Python gives an error when = is used instead of ==.

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# Truth Tables in Sage – using SymbolicLogic

Run SageMathCell at `https://sagecell.sagemath.org`:

```
sage: logic = SymbolicLogic()
sage: s = logic.statement("a&b")
```

Instead of `and`, use the `&` operator.

```
sage: t = logic.truthtable(s)
sage: logic.print_table(t)

a     | b     | value |
-----------------------
False | False | False |
False | True  | False |
True  | False | False |
True  | True  | True  |
```

# Truth Tables continued – the or operation

Sage session continued...

```
sage: s = logic.statement("a|b")
```

Instead of `or`, use the `|` operator.

```
sage: logic.print_table(logic.truthtable(s))
a     | b     | value |
-----------------------
False | False | False |
False | True  | True  |
True  | False | True  |
True  | True  | True  |
```

# Truth Tables continued – the not operation

Sage session continued...

```
sage: s = logic.statement("!a")
```

Instead of `not`, use the `!` operator.

```
sage: t = logic.truthtable(s)
sage: logic.print_table(t)
a     | value |
------------------
False | True  |
True  | False |
```

# Proving De Morgan's Law – with truth tables

The first law of De Morgan:

```
not (( not x ) or ( not y )) = x and y
```

Sage session continued ...

```
sage: law = '!((!x)|(!y))'
sage: s = logic.statement(law)
sage: logic.print_table(t)
x     | y     | value |
-----------------------
False | False | False |
False | True  | False |
True  | False | False |
True  | True  | True  |
```

We recognize the truth table for `x and y`.

# Summary + Assignments

In this lecture we covered

- section 1.1 in *Computer Science: an overview*
- sections 2.6.1 to 2.6.3 of *Python Programming*

Assignments:

1. Omit the brackets in De Morgan's Laws and create a truth table to evaluate the expressions.
2. Draw a flowchart for the code using a nested if else for the followup questions `"happy ?"` and `"bored ?"`.
3. Write pseudocode and draw of flowchart for a program that reads in a positive number and prints out whether the number is divisible by 2, 3, 5, or not.
4. Define a Python dictionary for the truth table of `x and y`.