Exam 1 MCS 275 Jan Verschelde Friday 22 February 2008

NAME :

Open book, open notes, but please do not ask questions.
Write all answers on these sheets.

question	1	2	3	4	5	total
points						
maximum	15	20	15	25	25	100

1. Write a Python function `BiDiag` which takes on input a positive number `n` and returns an n-by-n matrix A. All diagonal elements of A are 2 and all elements just above and below the diagonal are 1:

$$A = \begin{bmatrix} 2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 2 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 2 \end{bmatrix}.$$

`BiDiag` returns A as a two dimensional `numpy` array.

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2. Write a Python function which removes all duplicate elements of a list given on input. Call this function `RemoveDuplicates`.
If `L = [1, 3, 1, 4, 3, 3, 2]`, `RemoveDuplicates` will return `[1, 4, 3, 2]`.

   (a) Write an ***iterative*** version of `RemoveDuplicates`.

   (b) Write a ***recursive*** version of `RemoveDuplicates`.

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3. Apply divide and conquer to compute the sum of all numbers in a list: the total sum is the sum of the first half and the sum of the second half. Give a recursive function `RecSum` ***using divide and conquer*** which returns the sum of a list given on input.

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4. The Cantor set is defined by removing the middle third of [0,1] and then removing the middle third of the remaining intervals. The n-th Cantor set is obtained by executing the recursive removal n times. Cantor sets for n = 0, 1, and 2 are below:

```
n = 0 : [0,1]
n = 1 : [0,1/3], [2/3,1]
n = 2 : [0,1/9], [2/9,1/3], [2/3,7/9], [8/9,1]
```

(a) Write a function that returns the total length of all intervals which have been removed to form the n-th Cantor set. Complete the function definition below:

```
def LengthCut(n,a,b):
   """
   Returns the total length of the intervals removed
   from the interval [a,b] to form the n-th Cantor set.
   """
```

(b) Write a function that returns the lists of intervals in the n-th Cantor set. Complete the function definition below:

```
def CantorSet(n,a,b,L):
   """
   Returns the list of intervals for the n-th Cantor set.
   The list is accumulated in L.  In the first call L is [].
   """
```

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5. We use a binary tree to store a frequency table of words. The data at a node in the tree is a tuple like `(w,n)`, where the number `n` is the frequency of the string `w`.

   The binary tree is ordered: all words less than the word at a node in the tree are in the left branch while all other words are in the right branch of the tree.

   The tree `T` is represented as a recursive triple of triplets: as `(left,(w,n),right)` where left and right are again trees. The empty tree is the empty tuple `()`.

   (a) Give a Python function `LookUp` that given a tree and a word returns the corresponding frequency count stored in the tree. If the word does not occur in `T`, zero must be returned. Write a ***recursive*** version of `LookUp` below.

   (b) ***Use a stack*** to write an ***iterative*** version of the recursive `LookUp`.

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