Recursive Data Structures

MCS 275 Lecture 10
Programming Tools and File Management
Jan Verschelde, 6 February 2008

Binary Trees
sorting numbers
a recursive tree builder
flattening and searching
Classification Trees
lists and dictionaries
classifying with dictionary tree
adding elements to the tree
Exercises and Assignments

Recursive Data Structures

Sorting Numbers using a Tree

Consider the sequence 4, 5, 2, 3, 8, 1, 7

Insert the numbers in a tree:

Rules to insert x at node N:

- if N is empty, then put x in N
- if $x < N$, insert x to the left of N
- if $x \geq N$, insert x to the right of N

Recursive printing: left, node, right sorts the sequence.

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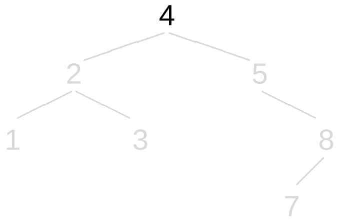

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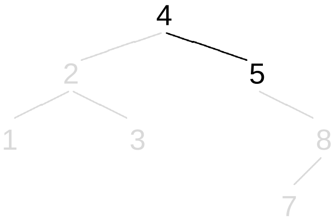

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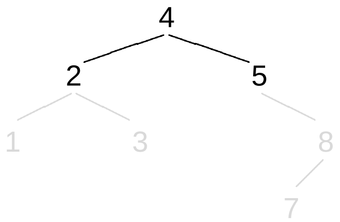

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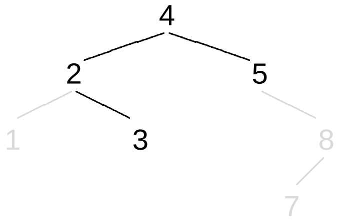

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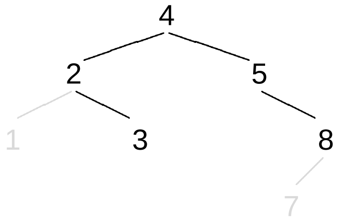

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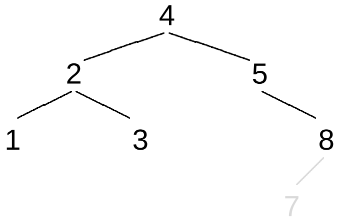

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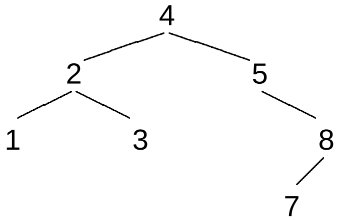

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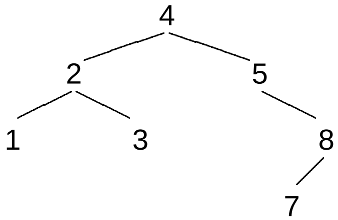

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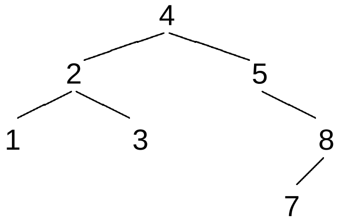

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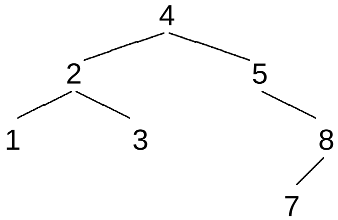

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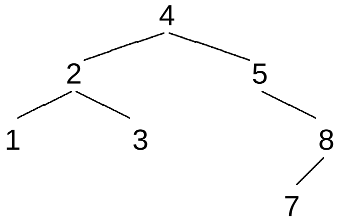

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Recursive Triplets in Python

Any node in a tree T is either empty or consists of

1. left branch (or child) is a tree
2. the data at the node is a number
3. right branch (or child) is a tree

To represent

we use tuples, sequences enclosed by (and):

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>>> L = ((),2,())
>>> R = ((),5,())
>>> T = (L,4,R)
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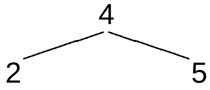

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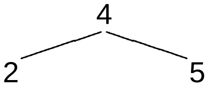

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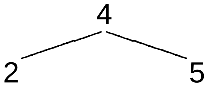

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Incremental Sorting

```
$ python treesort.py
give number (-1 to stop) : 4
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give number (-1 to stop) : 5
T = ((), 4, ((), 5, ()))
give number (-1 to stop) : 2
T = (((), 2, ()), 4, ((), 5, ()))
give number (-1 to stop) : 3
T = (((), 2, ((), 3, ())), 4, ((), 5, ()))
give number (-1 to stop) : 8
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give number (-1 to stop) : -1
sorted numbers = [2, 3, 4, 5, 8]
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Recursive Data Structures

Adding to the Tree

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def Add(T,n):
    """
    Adds a number n to the triple of triples.
    All numbers less than T[1] are in T[0].
    All numbers greater than or equal to T[1]
    are in T[2].  Returns the new tree.
    """
    if len(T) == 0:
       return ( () , n , () )
    elif n < T[1]:
       return ( Add(T[0],n) , T[1] , T[2] )
    else:
       return ( T[0] , T[1] , Add(T[2],n) )
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Flattening the Tree

The tree T

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(((), 2, ((), 3, ())), 4, ((), 5, ((), 8, ())))
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already orders the numbers increasingly, as

```
L = [2, 3, 4, 5, 8]
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To flatten the tree T into the list L,
we traverse T as follows:

- if the node is empty, return `[]`
- for a node that is not empty:
  1. let L be the flattened left branch
  2. append to L the data at the node
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  and finally return the list L

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def Flatten(T):
    """
    T is a recursive triple of triplets.
    Returns a list of all numbers in T
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    """
    if len(T) == 0:
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Searching the Tree

Suppose we do not wish to store duplicate elements.

To see whether a number n already belongs to a tree T we apply the following recursive algorithm:

- if T is empty, we return `False`
- if the data at the node is n, return `True`
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       return True
    elif n < T[1]:
       return IsIn(T[0],n)
    else:
       return IsIn(T[2],n)
```

the Function IsIn

```
def IsIn(T,n):
    """
    Returns True if n belongs to the tree,
    returns False otherwise.
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    if len(T) == 0:
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    elif T[1] == n:
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```

The Main Program

```
def main():
    """
    Prompts the user for numbers and sorts,
    using a tree: a triple of triplets.
    """
    T = ()
    while True:
        n = input('give number (-1 to stop) : ')
        if n < 0: break
        if IsIn(T,n):
            print n , 'is already in T'
        else:
            T = Add(T,n)
        print 'T =', T
    print 'sorted numbers =', Flatten(T)
```

The Main Program

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Binary Trees
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Trees with Lists and Dictionaries

To build trees with a variable number of children at each node we can use lists instead of tuples.

A more flexible indexing mechanism is provided by dictionaries (similar to `struct` in C).

For example, a mileage table to store distances from Chicago to Miami, Los Angeles and New York:

```
>>> mt = { 'Miami':1237 , 'LA':2047 , 'NY':807 }
>>> mt['LA']
2047
>>> mt.keys()
['Miami', 'NY', 'LA']
>>> mt.values()
[1237, 807, 2047]
```

A dictionary is a set of `key:value` pairs.

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Classifying Animals

Suppose we want to classify animals with simple questions with yes or no answers.

The leaves of the tree are just strings.

The internal nodes have questions as strings, and yes and no branches leading to more questions or to the names of the animals.

Classifying Animals

Suppose we want to classify animals with simple questions with yes or no answers.

Is it an insect ?

yes no

Does it fly ? It lives on a farm ?

yes no yes no

bee ant cow tiger

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Classifying Animals

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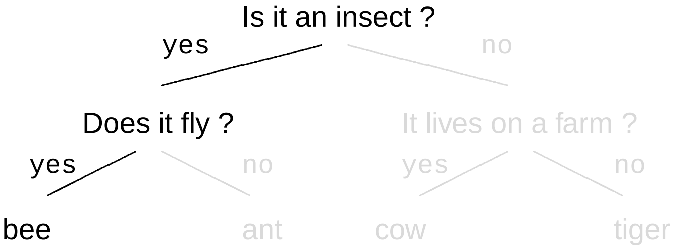

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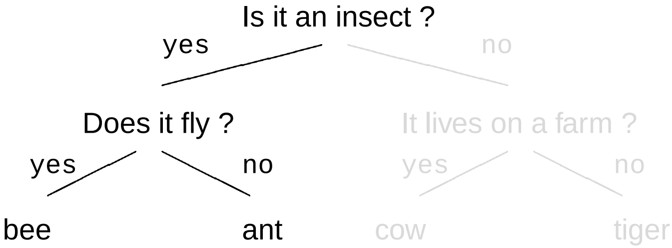

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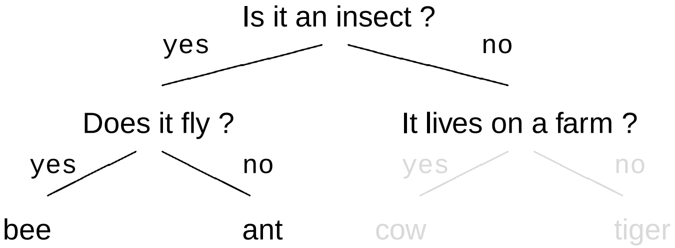

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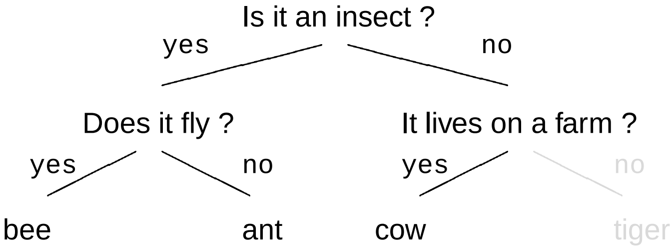

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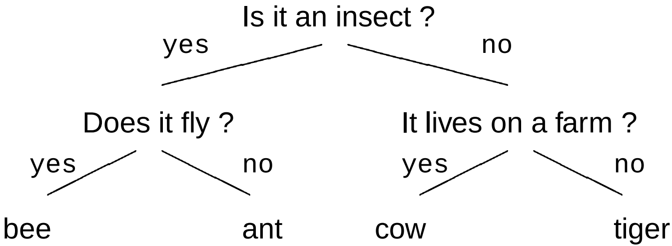

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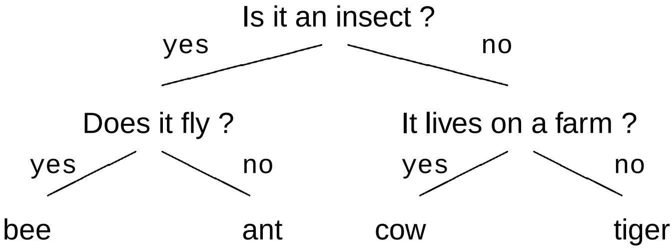

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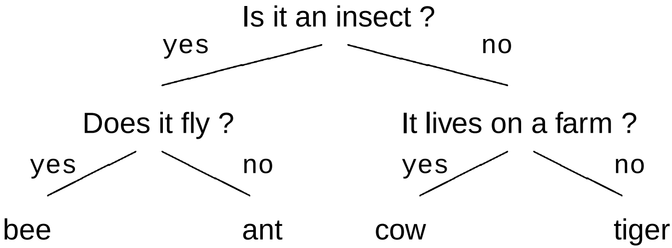

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Classifying with Dictionary Tree

```
$ python treezoo.py
What animal ? tiger
d = ['tiger']
continue ? (y/n) y
Is it "tiger" ? (y/n) n
What animal ? ant
Give question to distinguish "ant" from "tiger":
Is it an insect ?
d = {'q': 'Is it an insect ?', \
     'y': ['ant'], 'n': ['tiger']}
```

- Leaves in the tree are lists of one string.
- An internal node contains three keys:
  for the question, the yes and the no answer.

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```

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- An internal node contains three keys:
  for the question, the yes and the no answer.

Navigating through the Tree

Continuing with the script `treezoo.py`:

```
ended construction, start navigation...
Is it an insect ? (y/n) y
Does it fly ? (y/n) y
arrived at "bee"
```

The answer of the user `'y'` or `'n'`
is the key to the branches of the tree.

Navigation algorithm:

- base case: length of dictionary is one
- general case: follow answer of user

Navigating through the Tree

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Navigation algorithm:

- base case: length of dictionary is one
- general case: follow answer of user

the Function Navigate

```
def Navigate(d):
   """
   Navigates through the dictionary
   based on the user responses.
   """
   if len(d) == 1:
      print 'arrived at \"' + d[0] + '\"'
   elif len(d) == 3:
      a = raw_input(d['q'] + ' (y/n) ')
      Navigate(d[a])
```

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Recursive Data Structures

Adding Elements

Two base cases:

- for empty tree, we ask for animal name
- at leaf, we ask if it is the animal
  1. if yes, we are done
  2. if no, we ask name of new animal
     and a question to distinguish it

In the general case, we ask the question at the node and make a recursive call, adding to the branch `'y'` or `'n'`.

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In the general case, we ask the question at the node and make a recursive call, adding to the branch `'y'` or `'n'`.

the Function Add, part I


```
def Add(d):
   """
   Adds a new element to the dictionary,
   via interactive questions to the user.
   """
   if len(d) == 0:
      a = raw_input('What animal ? ')
      return [a]
   elif len(d) == 1:
      q = 'Is it \"' + d[0] + '\" ? (y/n) '
      a = raw_input(q)
      if a == 'y':
         print 'okay, got it'
         return d
```

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      a = raw_input(q)
      if a == 'y':
         print 'okay, got it'
         return d
```

the Function Add, part II

```
   else:
      a = raw_input('What animal ? ')
      s = 'Give question to distinguish \"' + \
          a + '\" from \"' + d[0] + '\":\n'
      q = raw_input(s)
      return {'q':q,'y':[a],'n':[d[0]]}
else:
   a = raw_input(d['q'] + ' (y/n) ')
   if a == 'y':
      return {'q':d['q'],'y':Add(d['y']),'n':d['n']}
   else:
      return {'q':d['q'],'y':d['y'],'n':Add(d['n'])}
```

the Function Add, part II

```
    else:
       a = raw_input('What animal ? ')
       s = 'Give question to distinguish \"' + \
           a + '\" from \"' + d[0] + '\":\n'
       q = raw_input(s)
       return {'q':q,'y':[a],'n':[d[0]]}
else:
    a = raw_input(d['q'] + ' (y/n) ')
    if a == 'y':
       return {'q':d['q'],'y':Add(d['y']),'n':d['n']}
    else:
       return {'q':d['q'],'y':d['y'],'n':Add(d['n'])}
```

The Main Program

```
def main():
    """
    Build a tree to classify animals.
    """
    d = {}
    while True:
        d = Add(d)
        print 'd =', d
        a = raw_input("continue ? (y/n) ")
        if a != 'y': break
    print 'ended construction, start navigation...'
    while True:
        Navigate(d)
        a = raw_input("continue ? (y/n) ")
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   print 'ended construction, start navigation...'
   while True:
      Navigate(d)
      a = raw_input("continue ? (y/n) ")
      if a != 'y': break
```

Exercises and Assignments

1. Use the code `Add`, `Flatten`, and `IsIn` as methods in a class to represent trees to sort numbers.
2. Modify the representation of the tree to sort numbers, using a dictionary instead of a triplet. As keys use the strings `'data'`, `'smaller'`, and `'larger'`. The leaves of the tree have only one element: `'data':number`.
3. For the tree of dictionaries to classify animals, write a function which takes on input the tree and returns the list of all animal names in the tree.

Homework collected on Friday 8 February:

- exercises 2 and 3 of Lecture 8,
- exercises 2 and 5 of Lecture 9,
- exercise 2 of Lecture 10.