

2. Consider the point $(1, 1)$ on the curve $f(x, y) = x^2 - y^3 - x + y = 0$.
- (a) Give the Maple command(s) to compute a Taylor series about the point $(1, 1)$ where the term of the error is of second order.
 - (b) Compute the slope of the tangent line of the curve at the point $(1, 1)$ and use the slope to determine the tangent line. Write the equation of the tangent line.

Verify that the tangent line equals the first-order Taylor series at the point $(1, 1)$.

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3. Consider the curve defined by the equation $f(x, y) = (x^2 + y^2)^5 - 16x^2y^2(x^2 - y^2)^2 = 0$.
- (a) Give the command to make a plot of this curve, for x and y in the range from -1 to $+1$. How many points do you need to obtain a good plot?

- (b) Give the commands to transform this curve into polar coordinates and to plot the curve. How many times does the curve pass through $(0, 0)$?

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4. What is a remember table in Maple?
Explain its use and how you can create a remember table.

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5. The k th Lagrange polynomial in z for n points x_1, x_2, \dots, x_n is defined as

$$L[x, k, n](z) = \frac{(z - x_1) \cdots (z - x_{k-1})(z - x_{k+1}) \cdots (z - x_n)}{(x_k - x_1) \cdots (x_k - x_{k-1})(x_k - x_{k+1}) \cdots (x_k - x_n)}$$

So $L[x, k, n](x[k]) = 1$ and $L[x, k, n](x[i]) = 0$ for all $i \neq k$. Write a procedure `L` which returns $L[x, k, n](z)$. For example: $L[x, 3, 4]$ returns $\frac{(z - x_1)(z - x_2)(z - x_4)}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)}$.

The argument of `L` is the symbol for the independent variable of the polynomial returned by `L`. Parameters to `L` are the symbol x , and numbers k and n .

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6. Consider the system
$$\begin{cases} f(x, y) = y - 5x(x - 1)(x - 2) = 0, \\ f(y, x) = x - 5y(y - 1)(y - 2) = 0. \end{cases}$$

- (a) Give the Maple commands to plot the curve defined by $f(x, y) = 0$ in blue and $f(y, x) = 0$ in red for x and y both in the interval $[-3, +3]$.
How many intersection points do you see?

- (b) Give the Maple commands to compute a triangular form of the system.
How many complex solutions does this system have? Justify your answer.

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7. A Hankel matrix of dimension n has first row $a[1], a[2], \dots, a[n]$.

The j th element on the i th row of the matrix equals $a[1 + ((i + j - 2) \bmod n)]$.

Give the Maple command to define a Hankel matrix H_5 of dimension 5.

How many terms does the determinant of H_5 have?

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