Maple Lecture 25. Three dimensional plots

In three dimensions we can plot surfaces and space curves.
Just as in the previous lecture, we start loading the plots package:

```
[> with(plots);
```

We cover some of [3, Chapter 15], see [1], [2], and [4] for more examples.

25.1 Explicit and Implicit Plots

Surfaces can be defined as $z = f(x, y)$.

```
[> f := (x,y) -> cos(x*y):
[> plot3d(f,-2*Pi..2*Pi,-2*Pi..2*Pi,axes=box);
```

If we click on the plot, we adjust the orientation and view the object from any angle.

Many surfaces are defined implicitly. For example, the Whitney umbrella is defined by the equation $x^2 - zy^2 = 0$.

```
[> umbfun := (x,y,z) -> x^2 - z*y^2:
[> umbrella := implicitplot3d(umbfun,-2..2,-2..2,-2..2,grid=[20,10,10]):
[> display(umbrella);
```

The problem with this plot is that the handle of the umbrella is missing. From the equation we see that $x = 0, y = 0$ is a solution for any value of z. But, we do not see the z-axis on the plot.

```
[> handle := plottools[line]([0,0,-2],[0,0,2]):
[> display(whitney,handle);
```

With this patch, we see that the umbrella is really not drawn very well... Maple does not consider the algebraic structure when plotting.

25.2 Options of plot3d

We can arrange the orientation to create a spin plot.

```
[> f := (x,y) -> x^3 - 3*x*y^2:
[> plotargs := f,-1..1,-1..1,axes=box:
[> plot3d(plotargs);
```

The orientation is described by two angles in spherical coordinates.

```
[> plot3d(plotargs,orientation=[45,45]);
```

As you can see, [45,45] is the default orientation. The first angle is the angle with the x-axis, we change this angle to turn the object to the left or right. The second angle is the angle with the z-axis. To see the object from above or below we change this axis.

```
[> spin := [seq(plot3d(plotargs,orientation=[10*i,45]),i=0..36)]:
[> display(spin,insequence=true);
```

As with two dimensional plots, the plotting happens in two stages: first the plot is computed and then rendered. This allows to make lists and sequences of plots, followed by the invocation of the display command.

25.3 Plotting Curves as Tubes

With tubeplot we make a tube around a space curve. Let us make a tubeplot from a circle.

```
[> x := cos(t): y := sin(t): z := 0:
[> torus1 := tubeplot([x,y,z],t=0..2*Pi,radius=1/4):
[> display(torus1);
[> torus2 := tubeplot([x+0.8,z,y],t=0..2*Pi,radius=1/4):
[> display([torus1,torus2]);
```

Let us make a chain of 10 tori :

```
[> chain := []:
[> optstube := t=0..2*Pi,radius=1/4:
[> dx := 0:
[> for i from 1 to 10 do
 >    if i mod 2 = 0
 >      then torus := tubeplot([2*x+dx,2*y,z],optstube):
 >      else torus := tubeplot([2*x+dx,z,2*y],optstube):
 >    end if:
 >    chain := [op(chain),torus]:
 >    dx := dx + 3:
 > end do:
[> display(chain,scaling=constrained);
```

25.4 Data plotting

We can make histograms from matrices:

```
[> indfun := (i,j) -> max(sin(i)*cos(j),0);
[> city := matrix(20,20,indfun):
[> matrixplot(city,heights=histogram);
```

25.5 Animation

Here we will make an animation of a torus knot:

```
[> r := 2 + 4/5*cos(7*t): z := sin(7*t):
[> curve := [r*cos(4*t), r*sin(4*t), z]:
[> picts := [seq(tubeplot(curve,t=0..2*Pi*i/20,radius=1/4),i=1..20)]:
[> display(picts,insequence=true,style=patch);
```

25.6 Riemann Surfaces

The logarithm for complex arguments defines a four dimensional surface: we let the real and imaginary part of the argument correspond to the usual x and y axes of three space. The vertical z-axis represents the real part of the function value, while the coloring of the surface is a function of the imaginary part.

```
[> w := u + I*v;
[> z := evalc(exp(w));
```

Observe how **evalc** expresses `z` as a function of `u` and `v`. As `z` equals `exp(w)`, `w` equals the natural logarithm of `z`. So we take `x` and `y` to be real and imaginary parts of `z`:

```
[> x := evalc(Re(z));
[> y := evalc(Im(z));
```

Then we plot the values for the complex logarithm in two ways, depending on what we choose as height and coloring.

First take the height to be the real value `u` of `w` and then take the coloring as the imaginary value `v` of `w`:

```
[> plot3d([x,y,u],u=-4..1,v=-3*Pi..3*Pi,orientation=[-56,72],colour=v);
```

Alternatively, we take height to be `v` and color with `u`:

```
[> plot3d([x,y,v],u=-4..1,v=-3*Pi..3*Pi,orientation=[-56,72],colour=u);
```

For more about complex plots, we refer to [1].

25.7 Assignments

1. A ski hill was designed in [2] using

$$h = 2\cos(0.4x)\cos(0.4y) + 5xye^{-(x^2+y^2)} + 3e^{-((x-2)^2+(y-2)^2)}.$$

   Choose appropriate ranges for x and y and adjust the orientation so you can clearly see three peaks.

   Can you add another peak?

2. Consider the surface defined by $z = \cos(x)\sin(y)$, for $x = -\pi \ldots \pi$ and $y = -\pi \ldots \pi$.

   Give all Maple commands to make a 3D animation of 10 frames that gradually flattens this surface, using the formula

$$\cos((1-0.1k)x)\sin(1-0.1k)y), \quad \text{for } k = 1, 2, \ldots, 10.$$

3. The *Viviani curve* is a space curve defined by $x = R\cos^2(t)$, $y = R\cos(t)\sin(t)$, and $z = R\sin(t)$, where R is some parameter.

   (a) Give the Maple commands to make a plot of this curve, for $R = 1$, using a tube around the curve of radius 0.05.

   (b) Make an animation for the curve $x = R\cos^2(t)$, $y = \cos(t)\sin(t)$, and $z = \sin(t)$, for R going from 1 to 10 (thus using ten frames). Give all Maple commands you use to make this animation.

4. Make an animation of a spiral, using the formulas $x = t\cos(t)$, $y = t\sin(t)$, and $z = t$. Let there be 30 frames in your animation, with $t = 1, 2, \ldots, 30$.

5. Consider $h := t \rightarrow (x^2+y^2-1)(1-t)+(x^2+y^2+z^2-1)t$ as a deformation of the cylinder $x^2+y^2-1 = 0$ at $t = 0$ to the sphere $x^2+y^2+z^2-1 = 0$ at $t = 1$.

   Use h to create an animation with 50 frames which deforms the cylinder into the sphere.

6. Consider `v := [[0,0,0],[1,0,0],[0,1,0],[0,0,1], [1,1,0],[1,0,1],[0,1,1],[1,1,1]]`.

   (a) Use `l12 := plottools[line](v[1],v[2],thickness=3,color=red)` to define the edges between the vertices in `v`. The cube has 12 such edges. Store all 12 edges in the sequence `edges`.

   (b) Display the `edges` with a perspective determined by `projection` equal to 0.8, no axes, and `orientation` equal to `[30,70]`.

7. Draw the Riemann surface for the $z^{1/3}$, defined by $w^3 - z = 0$, for $w = u + Iv$.

References

[1] R.M. Corless. *Essential Maple 7. An introduction for Scientific Programmers*. Springer-Verlag, 2002.

[2] R.H. Enns and G.C. McGuire. *Computer Algebra Recipes: A Gourmet's Guide to the Mathematical Models of Science*. Springer-Verlag, 2002.

[3] A. Heck. *Introduction to Maple*. Springer-Verlag, third edition, 2003.

[4] G. Klimek and M. Klimek. *Discovering Curves and Surfaces with Maple*. Springer-Verlag, 1997.