MCS 320 Project Three : simulating the n-body problem

The goal of this project is to use MATLAB or Octave to simulate the n-body problem from celestial mechanics. With simple scripts we will plot trajectories computed by solving a system of differential equations.

1. modeling with differential equations

We consider three bodies with respective masses m_1, m_2, m_3 in the plane with positions $(x_1(t), y_1(t))$, $(x_2(t), y_2(t))$, $(x_3(t), y_3(t))$ evolving over time t, governed by a system of second order differential equations, shown below for the movement of the first body:

$$\begin{aligned}
\frac{d^2x_1(t)}{dt^2} &= -\frac{m_2(x_1(t)-x_2(t))}{((x_1(t)-x_2(t))^2+(y_1(t)-y_2(t))^2)^{3/2}} - \frac{m_3(x_1(t)-x_3(t))}{((x_1(t)-x_3(t))^2+(y_1(t)-y_3(t))^2)^{3/2}} \\
\frac{d^2y_1(t)}{dt^2} &= -\frac{m_2(y_1(t)-y_2(t))}{((x_1(t)-x_2(t))^2+(y_1(t)-y_2(t))^2)^{3/2}} - \frac{m_3(y_1(t)-y_3(t))}{((x_1(t)-x_3(t))^2+(y_1(t)-y_3(t))^2)^{3/2}}
\end{aligned}$$

Four additional equations determine the positions for the second and third body. To turn this into a system of first order differential equations (as required for MATLAB (or Octave)) we introduce new variables u_i, v_i for the velocities of x_i, y_i so we have

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= u_1(t) \\
\frac{du_1(t)}{dt^2} &= -\frac{m_2(x_1(t)-x_2(t))}{((x_1(t)-x_2(t))^2+(y_1(t)-y_2(t))^2)^{3/2}} - \frac{m_3(x_1(t)-x_3(t))}{((x_1(t)-x_3(t))^2+(y_1(t)-y_3(t))^2)^{3/2}} \\
\frac{dy_1(t)}{dt} &= v_1(t) \\
\frac{dv_1(t)}{dt} &= -\frac{m_2(y_1(t)-y_2(t))}{((x_1(t)-x_2(t))^2+(y_1(t)-y_2(t))^2)^{3/2}} - \frac{m_3(y_1(t)-y_3(t))}{((x_1(t)-x_3(t))^2+(y_1(t)-y_3(t))^2)^{3/2}}
\end{aligned}$$

Applying this rewriting leads to a system of 12 first order differential equations in the positions and velocities of the three bodies. Defining the masses, initial positions and velocities leads to an initial value problem. Solving this initial value problem with `ode45` (in MATLAB) or with `lsode` (in Octave) leads to the trajectories of the bodies.

The first argument to the ODE solver is a function that evaluates the righthandside vector of the system of differential equations. At the course web site, the functions defined in the files `planar3rhsMatlab.m` and `planar3rhsOctave.m` evaluate the righthandside vector respectively for MATLAB and Octave. The scripts `planar3bodyMatlab.m` and `planar3bodyOctave` (respectively again for MATLAB and Octave) define the setup for the problem: initial values and range for t, call the solver, and then plot the trajectories.

The scripts plot the entire trajectory, for the requested time span. To make an animation, we select suitable ranges of the trajectories and perform the rendering in a for loop.

2. the assignments

The three assignments below can be solved independently from each other.

Assignment One: Generalize to 4 Bodies

The example is set up to work with three bodies. The goal of the first assignment is to generalize the model to four bodies.
Assignment One. Give your modified .m files to generalize the model to four bodies. Choose some good initial values to make a nice plot of the trajectories. Note that as the bodies collide, numerical problems may arise.

Assignment Two: Make a Spatial Model

Extend the model to three dimensions. To keep the complexity of the computations a bit under control, you may still work with three bodies, i.e.: solve this assignment independently of the previous assignment.
Assignment Two. Give your modified .m files to make a spatial model of the n-body problem. To plot the trajectories, use `plot3`. Make an animation of the evolution of the trajectories.

Assignment Three: a Stable Figure Eight Configuration

Take a look at the online paper [2] and check out the links this paper makes. The goal of this assignment is to make the figure eight configuration which has been used in space missions.
Assignment Three. Give the script to define the planar 3-body problem where the trajectories of the three bodies make a horizontal figure 8, as ∞. Also provide a print out of the plot.

3. the deadline is Friday 29 April 2011, at 11AM

Bring *your* solution to the project to class. The *your* is emphasized to stress that your solution is the result of an *individual* effort. Collaborations are **not** permitted.

Please do not mix MATLAB and Octave scripts. You are free to choose either MATLAB or Octave, but you may not solve one assignment with MATLAB and another with Octave. Every script must contain a line documenting whether it runs under MATLAB or Octave.
The solution to this project consists in two parts:
1. A print out of all the `.m` files that you bring to class.
In addition, make plots and a `.txt` file created with diary. Include a summary of your experiments, description of your observation and explanation of the results.
2. Email your scripts as an attachment to me so I can verify your runs.
If you have questions or difficulties, feel free to come to my office for help.

References

[1] Donald G. Saari. *Collisions, Rings, and Other Newtonian N-body Problems.* Number 104 of CBMS Regional Conference Series in Mathematics, A.M.S., 2005.

[2] Ivars Petersen. Strange Orbits. *Science News* 168(7), 2005.
`http://www.sciencenews.org/articles/20050813/mathtrek.asp`