Symbolic and Numeric Differentiation

1. Symbolic and Numeric Differentiation
   - symbolic differentiation
   - numeric differentiation

2. Implicit Differentiation
   - the tangent line at a point on a curve
   - implicit differentiation to compute the slope

MCS 320 Lecture 22
Introduction to Symbolic Computation
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Symbolic and Numeric Differentiation

Symbolic Differentiation

Example: compute the value of the derivative of $f = \sin(2\pi x)$ at $x = 1$.

This is a 2-step process:

1. $f'(x) = \dfrac{d}{dx}\sin(2\pi x) = \cos(2\pi x)2\pi$,
2. $f'(1) = 2\pi$.

Observe that the value in the example is a symbol.

We can describe this process in general as follows.

> Given an expression f which depends on a variable x, the result of *symbolic differentiation* is an expression f' for the derivative of f with respect to x.

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Numeric Differentiation

Given a function $f(x)$, and a value A for x,
the *central difference formula*

$$\text{for } h > 0 : \quad \frac{f(A+h) - f(A-h)}{2h} \approx f'(A)$$

approximates $f'(A)$, the value of the derivative of f at $x = A$.

The error of this approximation is $O(h^2)$.

Two concerns that are addressed by numerical analysis:

- The accuracy depends on a good choice for the step h, and the precision at which f is evaluated.
- Numerical extrapolation methods improve the accuracy.

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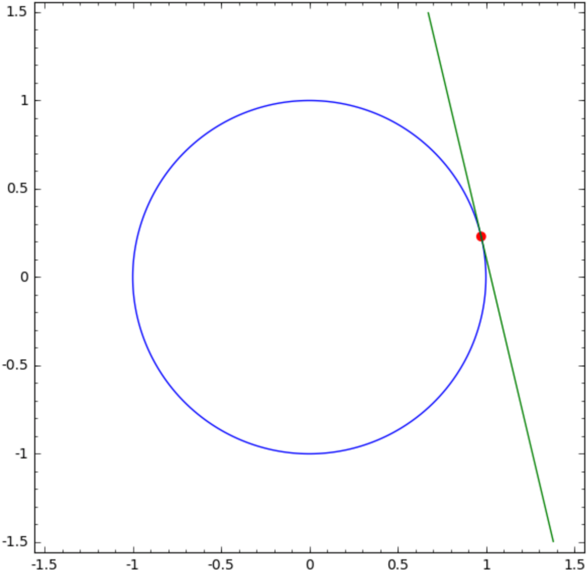
The Tangent Line at a Point on a Curve



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Implicit Differentiation to Compute the Slope

Input: $f(x,y) = 0$, an equation and a point $(a,b) : f(a,b) = 0$.

Output: $\left.\dfrac{dy}{dx}\right|_{(x,y)=(a,b)}$ is the slope of the tangent line to the curve $f(x,y) = 0$, at the point (a,b).

The three steps to compute the slope of the tangent line are below.

1. View $y = y(x)$ as a function of x.
2. Apply the operator $\dfrac{d}{dx}$ to the equation $f(x, y(x)) = 0$:
$$\frac{df}{dx} + \frac{df}{dy}\frac{dy}{dx} = 0.$$
3. Solve the equation for $\dfrac{dy}{dx}$.