Follow the instructions below:

1. The exam must be solved individually.
2. Submitting materials retrieved from the internet is plagiarism.
3. You may use all notebooks posted on the course web site and your own notebooks.
4. Solutions must be in a Jupyter notebook, with a SageMath kernel.
5. The questions are provided in a Jupyter notebook.
   You may use that notebook to formulate your answers to the questions.
6. Answers must be submitted before, or at 3:40pm.
7. Submit to gradescope.
8. Not submitting any answers will by default result in a zero score.
9. During the exam no questions will be answered, so do not ask questions.

Good Luck!

1. Let $N = \exp(\sqrt{5})$.
   (a) Compute a rational approximation for N, accurate to 32 decimal places.
   (b) Verify the accuracy of your rational approximation by computing the relative error. /15

2. Consider the finite field with 7 elements.
   Show that $p = x^3 + 3x^2 + 6$ is irreducible over this field.
   Declare α as a formal root of p. What is α^7 in this finite field extension? /10

3. Compute a numeric factorization of $p = x^4 + 3x^2 + x - 1$.
   Expand the factorization and compare the coefficients of the expanded form with the coefficients of p. What is the largest error on the coefficients? /10

4. Draw the binary expression tree defined by the fast callable object for
   $$(\cos(b) - \sin(c^2) + 2)/(bc - 2a).$$ /15

5. What does the preparser in SageMath do?
   Give an application of `preparse(x)`, with a good example for its argument `x`. /10

6. The `f = lambda n:  float(sum([(k/n)*exp(k/n) for k in range(1,n)])/n)` computes the right hand side of
   $$\int_0^1 x e^x dx \approx \frac{1}{n} \sum_{k=1}^{n-1} \left(\frac{k}{n} \right) \exp\left(\frac{k}{n} \right).$$
   (a) Time the execution of `f` for $n = 10000$. Explain why `f` is inefficient.
   (b) Apply vectorization to improve the efficiency. Verify the correctness.
   Time the execution of the vectorized function for $n = 10000$, compare with timings of `f`. /10

7. Let $q = (x^6 - 3x^4 + 2x + 1)/(x^6 + 2x^5 - 3x^4 + 3x + 7)$.
   Compute a partial fraction decomposition of q over the complex numbers.
   How many operations does it takes to evaluate q compared the number of operations to evaluate its partial fraction decomposition? /10

8. Transform $(x-y)^4 + \dfrac{(x+y)^3}{(x-y)^2}$ into $\dfrac{(x-y)^6 + (x+y)^3}{(x-y)^2}$, without retyping. /10

9. Are the expressions $p = \dfrac{x^2 + 7x - 8}{x - 1}$ and $q = x + 8$ the same?
   Justify your answer by appropriate *symbolic* computations.
   Demonstrate the application of a *numerical* probability-one equality test. /10