Follow the instructions below:

1. The exam must be solved individually.
2. Submitting materials retrieved from the internet is plagiarism.
3. You may use all notebooks posted on the course web site and your own notebooks.
4. Solutions must be in a Jupyter notebook, with a SageMath kernel.
5. The questions are provided in a Jupyter notebook.
   You may use that notebook to formulate your answers to the questions.
6. Answers must be submitted before, or at 3:40pm.
7. Submit to gradescope.
8. Not submitting any answers will by default result in a zero score.
9. During the exam no questions will be answered, so do not ask questions.

*Good Luck!*

1. Consider $f_k(x_1, x_2, \ldots, x_n) = x_k^3 + \prod_{i=1}^{k} x_i$. Define a function `F` which takes on input a list `X` of variables and an index `k` in $\{1, 2, \ldots, n\}$. `F(X, k)` returns $f_k(x_1, x_2, \ldots, x_n)$.

   Show that your function works for $n = 8$. /10

2. Use the recursion $s_n = 34 s_{n-1} - s_{n-2} + 2$, for $n > 1$, with $s(0) = 1$, $s(1) = 1$ to define a function `S` which takes on input $n$ and which returns $s_n$.

   Make sure your function can compute $s_{100}$. /15

3. Consider the point $P = (0, 1)$ on the curve $2x^2 - 3xy - 7y + 7 = 0$.

   Compute the slope of the tangent line to the curve at $P$. /10

4. Consider $f(x) = \displaystyle\int_x^3 \frac{1}{1-t} dt$.

   Use assumptions on $x$ to evaluate $f(x)$ for all cases of $x$. /10

5. Consider $p = x^5 - 2x^4 + 7x^3 + 4x^2 - x + 5$.

   (a) Compute a symbolic-numeric factorization of $p$.

   (b) Expand the factorization and compare the coefficients of the expanded form with the coefficients of $p$. What is the sum of the errors on all coefficients?

   /10

6. Plot the curve defined by $4x^3y^3 + 2x^5 - 2y^5 - x^2y^2 = 0$, for $x$ and $y$ in $[-1, +1]$.

   (a) Convert into polar coordinates and plot.

   (b) Describe the difference between the two plots.

   /15

7. Setup the system that must be solved to compute the point on the surface $x^2 + 2y^2 + 3z^2 = 5$ that is closest to the point $(1, 0, 0)$.

   Find the closest point among the solutions of the system. /10

8. Define the second order initial value problem

$$\frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) + 5y(t) = 50\cos(t), \quad y(0) = 0, \quad y'(0) = 0.$$

   Compute a symbolic solution for this problem. Plot the solution. /10

9. Minimize $8x_1 + 6x_2 + 36x_3$

   subject to $x_1 + 3x_3 \geq 3$, $x_2 + 4x_3 \geq 5$, and $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ .

   (a) What are the coordinates of the minimum?

   (b) What is the value of the target at the minimum?

   /10