Optimization

1. Polyhedral Optimization
   - geometry, algebra, business
   - the simplex method

2. Unconstrained Optimization
   - local and global optima
   - gradient descent

MCS 320 Lecture 32
Introduction to Symbolic Computation
Jan Verschelde, 17 July 2024

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Polyhedral Optimization

Viewed from a geometry, algebra, or business perspective:

- How far can we push a plane so it touches a polyhedron?
- Optimize a linear function subject to linear inequalities.

  Two normal forms:

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \end{array} \qquad \begin{array}{ll} \text{minimize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \geq \mathbf{b} \end{array}$$

- Minimizing cost and maximizing profit are equivalent in the polyhedral world.

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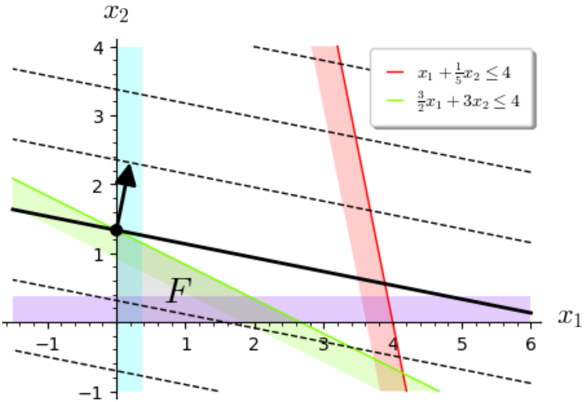
Linear Programming with the Simplex Method

Maximize $x_1 + 5x_2$ subject to $x_1 + 0.2x_2 \leq 4$ and $1.5x_1 + 3x_2 \leq 4$.



Starting at $(x_1 = 0, x_2 = 0)$, the simplex method pivots to
an adjacent corner of the feasible region that has a better optimum.

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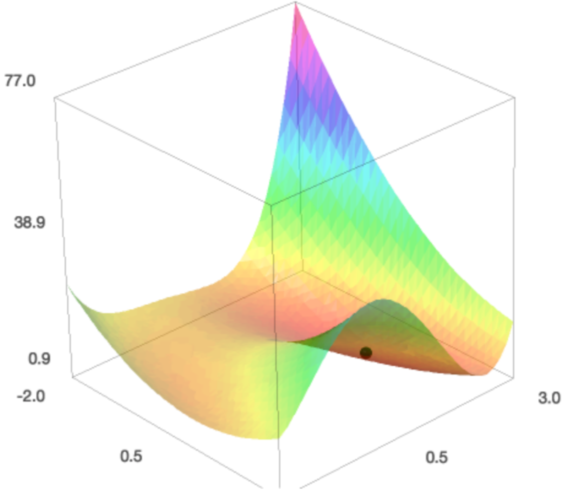
Unconstrained Optimization

Example: minimize $z = f(x, y) = (3 + x - y^2)^2 + (x - 1)^2 + (y - 1)^2$.



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Gradient Descent

For a function f in several variables, at the current point, the gradient descent method

1. evaluates the gradient ∇f at the current point,
2. updates the current point with a step in the ∇f direction.

The gradient descent method can find only a local optimum.

To find all optima,

1. compute all critical points, *solve* $\nabla f = 0$,
2. classify the points by their values in f.