Representation of Expressions

1. Expressions
   - operators and operands of expressions
   - expression trees are not binary
2. Evaluation
   - numeric and symbolic evaluation
   - binary expression trees for numerical evaluation

MCS 320 Lecture 13
Introduction to Symbolic Computation
Jan Verschelde, 26 June 2024

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Operators and Operands of Expressions

A polynomial p with m terms, n variables, coefficients in K:

$$p = \sum_{i=1}^{m} c_j \prod_{j=1}^{n} x_j^{d_{i,j}} \in K[x_1, x_2, \ldots, x_n].$$

Three operators:

1. $\sum_{i=1}^{m}$ represents addition, for i from 1 to m.
2. $\prod_{j=1}^{n}$ represents multiplication, for j from 1 to n.
3. The exponentiation is in the raised superscripts, in $x_j^{d_{i,j}}$, $d_{i,j}$ is the power of x_j in the i-th monomial.

Exponentiation is binary, addition and multiplication have operand *lists*.

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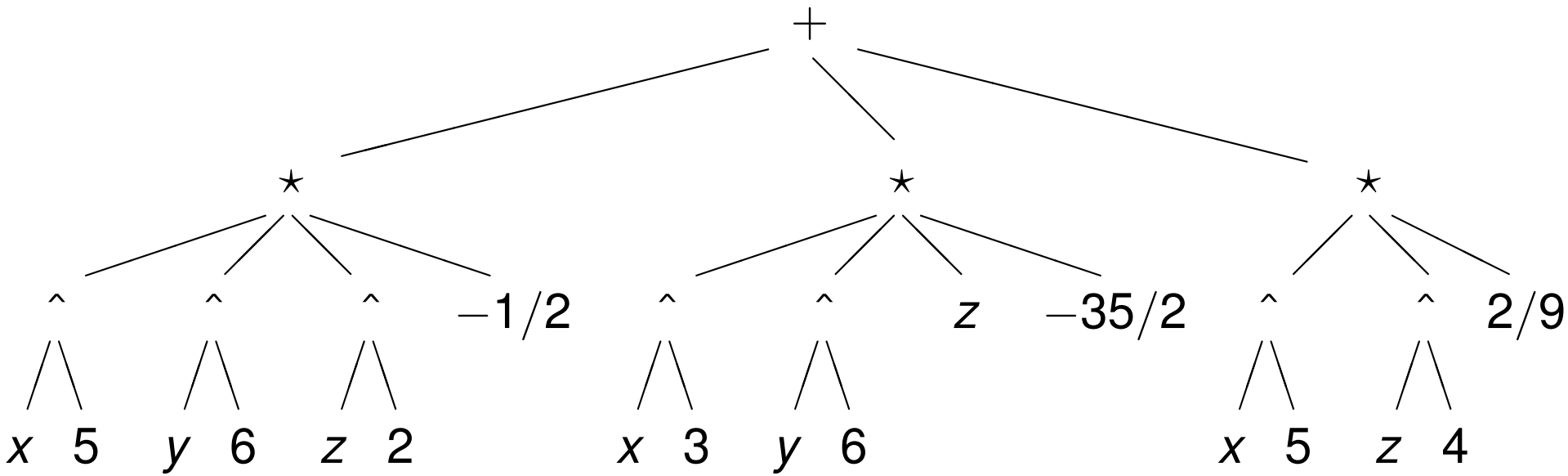
Expression Trees are not Binary

$$p = -\frac{1}{2}x^5y^6z^2 - \frac{35}{2}x^3y^6z + \frac{2}{9}x^5z^4$$



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numeric and symbolic evaluation

We distinguish between numeric and symbolic evaluation.

- Numerical evaluation:
  - The variables in an expression are replaced by numbers.
  - The result is a number.
- Symbolic evaluation:
  - The variables in an expression are replaced by symbols.
  - The result is an expression.

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binary expression trees for numerical evaluation

The numerical arithmetical operations are binary.
Therefore, for numerical evaluation, the expression trees are binary.

For example, $-\frac{1}{2}x^5y^6z^2$ has as expression tree: