Symbols, Names, and References

1. Computing with Symbols
   - an example of a symbolic computation
   - the symbolic ring
2. Names and References
   - the assignment of names and values
   - strings to prevent evaluation of symbols

MCS 320 Lecture 6
Introduction to Symbolic Computation
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an example of a symbolic computation

$$
\begin{aligned}
\left(a+b\sqrt{2}\right)\left(c+d\sqrt{2}\right) &= a\left(c+d\sqrt{2}\right)+b\sqrt{2}\left(c+d\sqrt{2}\right) \\
&= ac+ad\sqrt{2}+b\sqrt{2}c+b\sqrt{2}d\sqrt{2} \\
&= ac+ad\sqrt{2}+bc\sqrt{2}+bd2 \\
&= \left(ac+2bd\right)+\left(ad+bc\right)\sqrt{2}
\end{aligned}
$$

- The letters a, b, c, and d are symbols.
- As symbols, we compute with a, b, c, and d as with numbers.
- $a+b\sqrt{2}$ and $c+d\sqrt{2}$ are symbolic expressions, they are not algebraic numbers in $\mathbb{Q}(\sqrt{2})$.
- The last step in the calculation is a *normalization*.

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the Symbolic Ring

Why does $\sqrt{x^2}$ not automatically simplify to x?

There are automatic simplifications, for example:

- $\sqrt{4}$ simplifies to 2
- $\sin(\pi)$ simplifies to 0

To prevent simplification in the examples above,

1. numbers must be treated as symbols,
   or be cast into the symbolic ring;
2. the function must be told to hold off the evaluation.

Holding off an evaluation is delaying the evaluation,
till some point in the future when the hold is removed.

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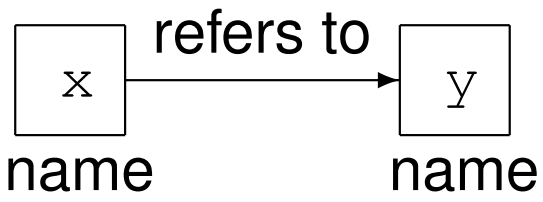
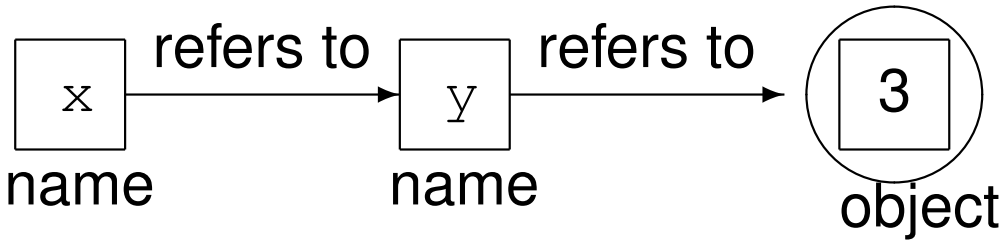
the Assignment of Names and Values

Let x and y both be the names of variables.

1. After assigning y to x, we have the following picture:



2. After assigning 3 to y, the situation is as follows:



The consequences of this arrangement:

- Evaluating y gives 3. Evaluating x to the fullest also gives 3.
- Assignments to y also change the value of x.

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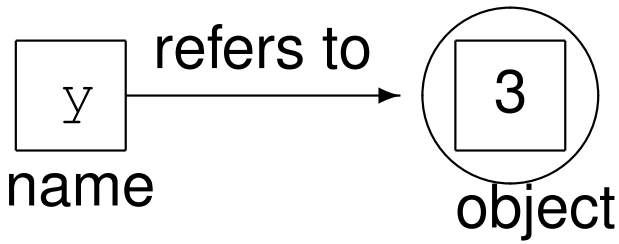
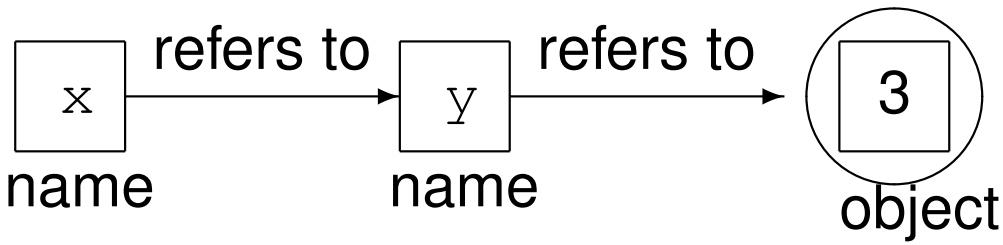
preventing the evaluation of symbols

Consider the following scenario.

1. We assigned 3 to the variable *y*, so we have:



2. Now, assign *y* to the variable *x*, so that this situation holds:



How do we prevent the evaluation of *y* to 3, when we assign *y* to *x*?