Review of the lectures 15 to 28

The exam is open book, open notes and open computer. To prepare for the exam you must organize your course materials to be ready for fast consultation. The worksheets of the lectures will still be available on the web for browsing; however, there is no guarantee the network or the computers at the math department will function properly. A diligent student has a backup on a zip disk and/or on paper.

Typically, on an open book exam, you use the course materials for consultation (e.g., to seek confirmation for your methods), not for study. If you have no clue how to start on a problem, you will run out of time if you start scanning all course materials looking for some hint.

The questions below are just samples of the type of questions you may expect. Also review the homework assignments and quizzes. The experience gained in the project may also help you.

1. Use rand() to generate a list of 100 numbers. Call the list `l`.
   (a) Replace every element x in the list `l` by x mod 5.
   (b) Remove all duplicates from the list `l`.

2. Explain why piecewise is often preferable to a similar if-then-else instruction.
   Give a good example to illustrate your explanation.

3. Write an indexed procedure with name **rf**, which returns **rf**$[n](x)$, defined by

   $$\mathbf{rf}[0](x) = 1,\ \mathbf{rf}[1](x) = x, \text{ and } \mathbf{rf}[n](x) = (x+1)(\mathbf{rf}[n-1](x) - \mathbf{rf}[n-2](x)), \text{ for } n \geq 2.$$

   The index n is the degree of the polynomial. Make sure the recursion runs efficiently.

4. Divided differences for a function $f(x)$ are defined as follows:

   $$f[x_1, x_2, \ldots, x_{n-1}, x_n] = \frac{f[x_2, \ldots, x_{n-1}, x_n] - f[x_1, x_2, \ldots, x_{n-1}]}{x_1 - x_n}, \quad \text{for } n > 1$$

   and $f[x_k] = f(x_k)$, for all k.

   Write a Maple procedure `dvd` which computes divided differences for any f, and is called like `dvd[a,b,c,d](f)`. The output of `dvd[a,b,c,d](f)` shows

   $$\frac{\dfrac{\frac{f(d)-f(c)}{c-d} - \frac{f(c)-f(b)}{b-c}}{b-d} - \dfrac{\frac{f(c)-f(b)}{b-c} - \frac{f(b)-f(a)}{a-b}}{a-c}}{a-d}.$$

   For simplicity, assume the user always makes the correct call to `dvd`, i.e.: include no error handling features.

5. What is a remember table in Maple? How is it used?
   Mention an example of a good use of a remember table.

6. Use the arrow operator to define the following operations on a polynomial p:
   (a) remove all terms with negative coefficients from p;

(b) replace x by x^2 in $p(x)$.

Use these two functions to define a function which does both operations to a polynomial.

7. Create a function in the variables B and N which returns $B \sum_{k=0}^{N} r_k^k$, where r_k is a random number drawn from a normal distribution with mean five and standard deviation $0.1k$.

8. Explain the difference between symbolic and automatic differentiation. Illustrate with an example the difference between the two and give the two Maple commands you need.

9. What is the difference between int and Int?
Give a good illustration why we need a command like Int.

10. Consider the function $f(t) = \int_0^t (1 - e^t) dt$, for $t \geq 0$. Define this function in Maple.
What is $f'(0)$?

11. The function $g(x,t) = \frac{1-t^2}{1-2xt+t^2}$ is a generating function for the Chebyshev polynomials.

    (a) Compute a Taylor series approximation for $g(x,t)$ around $t = 0$ of order 10. Select the coefficient of t^8 and compare with the output of `orthopoly[T](8,x)`. What is the difference between the two?

    (b) Make a function `cp` in n (n is the degree of the Chebyshev polynomial) which uses this generating function and returns the same expanded polynomial as the one returned by `orthopoly[T](n,x)`. The function `cp` should work for any n, be careful for $n = 0$.

12. Consider $p(x) = 5x^2a^2 + 61x^2a + 66x^2 + 10xa^2 + 121xa + 121x + a^2 + 15a + 44$, as a polynomial in x with parameter a.

    (a) Find the roots of p.

    (b) For which values of the parameter a is the answer valid?

    (c) Give the Maple command(s) to treat the special case(s).

    (d) As you can see the polynomial p is shown in expanded form. Give the Maple command to "un-expand", i.e.: what is the command which reveals better the structure of p?

13. How would you best solve for x the following expression:

```
            11             8              7              5             3
-42 sin(x)    + 88 sin(x)   - 76 sin(x)   - 65 sin(x)   + 25 sin(x)   + 28
```

14. Let a and b be positive numbers. Consider $f = \frac{x^2}{a} + \frac{y}{b}$ and the unit circle $x^2 + y^2 = 1$.
Give all Maple commands ...

    (a) to determine the number of extremal values of f on the unit circle.

    (b) to show how to compute one (only one!) such extremal value.

15. The logarithmic spiral is defined by $r = ae^{bt}$ in polar coordinates.

    (a) Give the Maple commands to make a plot for $a = 0.5$ and $b = 0.07$, for $t = 0 \ldots 6\pi$.

    (b) Create an animation of 10 frames, for $a = 0.5$ and for b going from 0.01 to 0.1 (also for $t = 0 \ldots 6\pi$).

16. Consider the curve $x^4 - 3xy + y^4$. Give all Maple commands

    (a) to make a plot for x and y both ranging between -2 and $+2$;

    (b) to convert the curve into polar coordinates; and

    (c) to plot the curve in polar coordinates.

17. Consider the system $\left\{ \begin{array}{r} 4 + 2x + 2y + z + xy = 0 \\ 4 + 2x + 2y + 2z + 4y^2 = 0 \\ x + y + x^2 = 0 \end{array} \right.$

    (a) How many complex solutions does the system have? Justify your answer.

    (b) Find all complex solutions of the system.

    (c) Suppose we were only interested in the rational or real solutions to the system. How would you modify your answer to the previous question to limit Maple to compute only the real or rational solutions?

18. Sometimes Maple displays symbols like _C1, _C2, etc... in its output.

    (a) What does this mean?

    (b) Give a good example of a problem which would show symbols like _C1, _C2, etc...

19. Consider the initial value problem

$$x''(t) + 4x(t) = sin(t), \quad x(0) = 1, x'(0) = 0.$$

    (a) Give all Maple commands to define this problem and to solve it numerically.

    (b) Define a function which returns for every t the value of $x(t)$.

    (c) Plot the solution for t going from 0 to 10.

20. Give the Maple commands for the following tasks.

    (a) Create a 5-by-5 matrix A where the (i, j)-the element is $\frac{1}{i+j}$ and a 5-by-1 matrix b of ones.

    (b) Construct the augmented matrix $[A\ b]$ in order to solve the system $Ax = b$ with Gauss-Jordan elimination, i.e.: use LinearAlgebra[ReducedRowEchelonForm].

    (c) Compute a LU Decomposition of the matrix A and use ForwardSubstitute and BackwardSubstitute from the LinearAlgebra package to solve $Ax = b$.

As this list is by no means exhaustive, you should review all homework assignments at the end of each lecture. Make sure you understand the solutions to the quizzes.

Please note the policy on skipping exams: If an exam is missed, then greater weight will be placed on the final exam, especially on the material covered on the missing exam. **What this means is** that if you decide not to take one midterm exam, your final exam will be weighted for one hundred points more. **What it does NOT mean is** that you can drop the score of a midterm exam. If you take the midterm, then your score counts. So, please be prepared when you show up for the exam.