

NAME :

**Open book, open notes, open computer, but closed mouth!**  
**Write all answers on these sheets.**

|          |    |    |    |    |    |    |    |       |
|----------|----|----|----|----|----|----|----|-------|
| question | 1  | 2  | 3  | 4  | 5  | 6  | 7  | total |
| points   |    |    |    |    |    |    |    |       |
| maximum  | 15 | 10 | 15 | 15 | 15 | 15 | 15 | 100   |

1. Give the Maple commands for the following tasks.

- (a) Use `stats[random,uniform[0,2*Pi]]` to generate ten random angles. With these angles, define ten random points on the unit circle  $x^2 + y^2 = 1$ , using  $x = \cos(a)$ ,  $y = \sin(a)$ , for every angle  $a$ . Store the points as a list of coordinates  $[x, y]$ , i.e., as  $[[x1, y1], [x2, y2], [x3, y3], \dots]$ .

- (b) With the list of points, make a list of plots of disks, centered at the ten random points and each with radius 0.01.

2. What is the assume facility in Maple. Why is it needed?

Give an example of a good use of the assume command.

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3. The Cantor set for any interval  $[a, b]$  is constructed by removing the middle third from the interval, leaving the two intervals  $[a, a + (b - a)/3]$  and  $[b - (b - a)/3, b]$ , and repeating this removal from the intervals that are left. Write a procedure `cantor` which is called with as index a natural number  $n$  and as arguments the endpoints  $a$  and  $b$  of an interval  $[a, b]$  (thus  $a < b$ ). Calling `cantor[n](a, b)` returns a sequence of  $2^n$  intervals, defined by the removal operation described above. For example, `cantor[0](0,1) = [0,1]`, `cantor[1](0,1) = [0,1/3],[2/3,1]`, `cantor[2](0,1) = [0,1/9],[2/9,1/3],[2/3,7/9],[8/9,1]`, etc.

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4. Consider the integral  $\int_0^t e^{\sin(x)} dx$ . Give the Maple commands

(a) to define this integral as a formula (call it **f**), without asking Maple to evaluate;

(b) to compute a 10-th order Taylor approximation (call it **T**) for it around  $t = 0$ ;

(c) to use the Taylor approximation to create a function **F** which returns a number, e.g.: **F**(2) returns 12496/2835.

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5. Consider the system 
$$\begin{cases} y^2 - xy + y + 1 = 0 \\ -2z^2 + y^2 + x = 0 \\ yz - z + 1 = 0 \end{cases}$$

Give all Maple commands needed to answer the following questions. Also, justify your answers.

(a) How many *complex* solutions does this system have?

(b) How many *real* solutions does this system have?

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6. The lemniscate of Bernoulli is defined by  $(x^2 + y^2)^2 = x^2 - y^2$ .  
Give all Maple commands used to convert the formula into polar coordinates.  
Give the formula in polar coordinates you obtain and the plotting command.

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7. Consider the initial value problem  $dy/dx = \sin(2x - y)$ ,  $y(0) = 0.5$ .
- (a) Give *all* Maple commands (not the output) to define this initial value problem and to obtain an **exact** solution.

- (b) Give the first three significant digits of  $y(1)$ : \_\_\_\_\_
- (c) Use the exact solution to create a function which gives a hardware float numerical approximation for every  $x$ . Give the Maple command(s) you used.

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