Maple Lecture 12. Representation of Expressions

In this lecture we take a detailed look at the internal representations of expressions. The material in this lecture is inspired on [1, Section 6.1 and 6.2]. See [2] for more specific details.

12.1 Internal representation of polynomials

There are three levels of selecting data from a polynomial. At the highest level we have coeffs and coeff. The commands nops and op work at a lower level. Finally, at the lowest level, we use dismantle to get to the Directed Acyclic Graph representation of a polynomial.

\[
\begin{align*}
\text{> } & p := x^{10} + 2 \cdot x^4 + 9; \\
\text{> } & \text{coeffs}(p); \quad \# \text{ sequence of coefficients} \\
\text{> } & \text{whattype}(p); \quad \# \text{ same as op(0,p)};
\end{align*}
\]

While \( p \) is a polynomial, it is also a general expression of type + on which the usual selectors work:

\[
\begin{align*}
\text{> } & \text{nops}(p); \quad \# \text{ number of terms} \\
\text{> } & \text{op}(p); \quad \# \text{ sequence of terms} \\
\text{> } & \text{op}(1,p); \quad \# \text{ select first term of p} \\
\text{> } & \text{op}([2,1],p); \quad \# \text{ select 1st term of 2nd term of p}
\end{align*}
\]

With the \texttt{op} command, we can construct the expression tree, shown in Figure 1.

![Figure 1: Expression tree of \( x^{10} + 2x^4 + 9 \).]

The expression tree is still very much a logical view on the polynomial. Internally, Maple stores an expression as a \textit{Directed Acyclic Graph}. From the output of \texttt{dismantle} we draw this graph in Figure 2.

\[
\begin{align*}
\text{> } & \text{dismantle}(p);
\end{align*}
\]

![Figure 2: Directed Acyclic Graph of \( x^{10} + 2x^4 + 9 \).]

Notice: in storing the terms in the sum, the monomials come first, followed by the coefficients. So the polynomial \( x^{10} + 2x^4 + 9 \) is actually stored like \( x^{10} \times 1 + x^4 \times 2 + 9 \times 1 \).
Now we understand the structure, and knowing that Maple stores every object only once, we can explain the effects of a substitution like this (see Figure 3):

```maple
> q := subs(1=Pi,p);
> dismantle(q);
```

![Directed Acyclic Graph of $x^{10} \pi + 2x^4 + 9 \pi$.](image)

Observe that `protected` is an attribute to the name `Pi`. When a variable has no attribute (like `x`), then this field of the name simply points to `NULL`.

Since the polynomial $p = x^{10} + 2x^4 + 9$ is stored like $x^{10} \times 1 + x^4 \times 2 + 9 \times 1$ and every object is stored only once, we now see why the command `subs(1=Pi,p)` creates the polynomial $x^{10} \times \pi + x^4 \times 2 + 9 \times \pi$.

### 12.2 Generalized rational expressions

Maple can work with polynomials where the variable is replaced by, for example a cosine:

```maple
> pcos := subs(x=cos(x),p);
> whattype(pcos); type(pcos,polynom);
```

While the expression is no longer a true polynomial, we can still ask for the coefficients:

```maple
> coeffs(pcos);
```

But we can no longer factor the polynomial directly:

```maple
> factor(pcos,complex);
```

We have to work indirectly, first factor $p$ and then substitute $x$ by $\cos(x)$ in the factored polynomial:

```maple
> fp := factor(p,complex);
> fpcos := subs(x=cos(x),fp);
```

General rational expressions are rational polynomial in disguise:

```maple
> rp := ((sin(y))^2-1)/(sin(y)-1);
> type(rp,ratpoly);
> factor(numer(rp));
> normal(rp);
```

It is interesting to see the internal structure (see Figure 4):

```maple
> dismantle(rp);
```

The `dismantle` procedure is one of the tools in the “hardware package” of Maple. Like we could do in a low-level programming language as C, we can manipulate addresses. For example:

```maple
> dismantle[dec](p);  # see the addresses in decimal notation
> addressof(p);  # verify the address of p
```
12.3 Assignments

1. For the polynomial \( p = x^3 - 8x + 1 \), give the Maple command (only one command!) to select the coefficient \(-8\). Draw the expression tree for \( p \) and the directed acyclic graph of its internal representation.

2. Use dismantle to investigate the internal representation of the polynomial \( 2x(y^2 + 1)^2 \). From the output of dismantle, draw the directed acyclic graph of this polynomial.

3. Consider the following instructions

   \[
   \begin{align*}
   &> x^2 - x + 1/x - 1/x^2; \\
   &> \text{subs(-1=1,%);}
   \end{align*}
   \]

   Explain the result of this substitution.

4. Compare the internal representations of \( p = x^8 + 2x^4 + 3 \) with \( q = (x^4 + 2)x^4 + 3 \).

   Draw the directed acyclic graphs of \( p \) and \( q \) based on the output of dismantle.

5. Do \texttt{dismantle[dec]}(p); after \( p := x^8 + 2x^4 + 3 \).

   Explain how you can see from the output of dismantle that the name \( x \) is represented only once.

References
