

NAME :

**Open book, open notes, but closed mouth!**  
**Write all answers on these sheets.**

question	1	2	3	4	5	6	7	total
points								
maximum	10	15	15	10	25	10	15	100

1. The multiplication table for  $\mathbb{Z}_7$  is

*	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

- (a) What is  $1/5$  in  $\mathbb{Z}_7$ ?
- (b) How can you see from the table that every element (except 0) in  $\mathbb{Z}_7$  has a multiplicative inverse?

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2. (a) Explain the difference between **evalhf** and **evalf**.

(b) When do we use **evalhf** and when must we use **evalf**?

(c) How can you force Maple to always use the hardware floats?

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3. Consider the sequence `restart; a := b; b := c; c := "d";`

(a) Draw a picture to illustrate the links between the variables, after the sequence was executed.

(b) Give the Maple commands to verify the links between the variables.

(c) Complete the sequence `restart; c := "d";` with the appropriate Maple command(s) to establish the same dependencies between the variables as the ones established by the sequence above.

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4. Give two different normal forms of a polynomial in several variables.

Illustrate these two normal forms on  $p = 2x + x^2 + 4xy + 3x^3 + x^2y^2 + 3y^4$ .

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5. Consider the output of `dismantle(p)`:

SUM(5)	
PROD(5)	(a) Draw the directed acyclic graph to show the internal representation of <code>p</code> .
NAME(4): <code>x</code>	
INTPOS(2): 1	(b) What does the expression <code>p</code> look like? Give the formula for <code>p</code> .
NAME(4): <code>y</code>	
INTPOS(2): 1	(c) Suppose we do <code>q := subs(1=Pi,p)</code> ; how many times will we see $\pi$ in the output? Justify your answer.
INTPOS(2): 1	
NAME(4): <code>x</code>	
INTPOS(2): 1	

Give the answers to the three questions below:

(a) Drawing:

(b) `p =`

(c) #times  $\pi$  in `q`:  
Justification:

6. Consider  $p = (x^4 + 1)(x^6 - 1)$ .

(a) Write instructions (like the ones produced by `codegen[C]`) to evaluate  $p$  in an optimal manner, i.e.: with as few arithmetical operations as possible.

(b) How many arithmetical operations does it take to evaluate  $p$ ?

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7. Explain the difference between symbolic and numerical factorization of a polynomial in one variable, referring to the number field used.

(a) Give an example where symbolic and numerical factorization are DIFFERENT. Give the polynomial and its symbolic and numerical factorization.

(b) Given a symbolic factorization, how can one obtain a numerical factorization fast in Maple?

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