Review of the first two parts of the course

In the first two parts of the course we learned about Maple’s advanced number system and about its favorite objects: polynomials and rational expressions.

As the final exam happens in a computer lab, all worksheets and lecture notes will still be available to you via the course web site. However, as there is always a slim chance that the network or web server fails, it is strongly recommended to have a well organized folder with lecture notes.

Below is a first, preliminary list of fresh questions to review. Consider also the review questions for the midterms, the quizzes, midterms, and homework assignments.

Review of Part One: First Steps with Maple

1. Explain the difference between \texttt{evalhf(1.0+10^{-10})} and \texttt{evalhf(1+10^{-10})}.

   The machine precision of a floating-point number system is defined as the smallest number you can add to one and obtain a result that is still larger than one. Give the Maple command(s) to get the magnitude of the machine precision.

2. Consider \( \mathbb{Z}_{31} = \{0, 1, 2, \ldots, 31\} \) and answer the following questions:
   (a) What is the multiplicative inverse of 7 in \( \mathbb{Z}_{31} \)?
   (b) Show that \( p = 15x^5 + 4x^4 + 23x^3 + 26x^2 + 6x + 1 \) is irreducible over \( \mathbb{Z}_{31} \).
   (c) Declare \( \alpha \) as a formal root of \( p \). How many elements has \( \mathbb{Z}_{31}(\alpha) \)? Justify your answer.
   (d) Compute the value \( \alpha^{21} \) as an element in \( \mathbb{Z}_{31}(\alpha) \).

3. Give all Maple commands to write \( e^{i \frac{2 \pi}{15}} \) as \( \cos(2 \frac{\pi}{15}) + i \sin(2 \frac{\pi}{15}) \).

4. The sequence \texttt{restart; s := a+b: a := x+y: b := u+v: s;} shows \( x+y+u+v \).
   (a) Give the Maple command to show that Maple still knows that \( s = a + b \).
   (b) Give one single Maple command to change \( s \) so that typing \( s \) shows \( x + y + u + v + c \).

5. Illustrate a good use of the \texttt{assign} command.
   Give an example of a Maple session in which the outcomes of \texttt{assign(x,5)} and \( x := 5 \) are different.

6. Consider the expression \( q = \cos(x^3 - 1) + 3 \sin(y) - z^7 \). Draw the expression tree for \( q \) and give all Maple commands you used to make the drawing.

7. How do we bring a matrix of floating-point numbers from file into a Maple session? Illustrate with a good example.

8. Generate optimized code to evaluate \( p = 79x^{298} + 56x^{205} + 49x^{164} + 63x^{121} + 57x^{119} - 59x^{42} \).
   How many arithmetical operations are needed to evaluate \( p \)? Compare with the cost of a direct evaluation of \( p \).
Review of Part Two: Polynomials and Rational Expressions

9. Draw the internal representation of \( p := xy(x - y) \). Give also the Maple command(s) (but not the output!) used to obtain your drawing. Explain why \( \text{subs}(1=-1,p) \) returns \( \frac{1}{xy(1 - y)} \).

10. Consider the polynomial \( p = x^3 - x - 2 \) and give all Maple commands following questions:
   (a) to write \( p \) as an **exact** product of linear factors, with **exact** complex numbers;
   (b) to compute a **numerical** factorization of \( p \) over the complex numbers;
   (c) to define a **symbolic** (i.e.: formal) factorization of \( p \), declaring sufficiently many roots.

11. Give all Maple commands to transform \((x - y)(x + y)\) into \((x + y)x - (x + y)y\).

12. Consider the rational expression \( r = \frac{79x^5 + 56x^4 + 49x^3 + 63x^2 + 57x - 59}{45x^5 - 82x^4 - 93x^3 + 43x - 62} \).
   Convert \( r \) into a form which is more efficient to evaluate. Compare the number of arithmetical operations needed to evaluate \( r \) in this more efficient form with the number of arithmetical operations needed to evaluate \( r \) in its given form.

13. Explain why normal forms are so important to symbolic computation.
   What can we do if a normal form is too expensive to compute? Illustrate with a good example.

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FINAL EXAM is in CCC 408 on Tuesday 4 May 2004
from 1:00 till 3:00PM if your last name begins with A to K; and
from 3:00 till 5:00PM if your last name begins with L to Z.

In case of a scheduling conflict with another final exam, please let me know as soon as possible so we can schedule a makeup.

Observe the university rules concerning incompletes. An incomplete can only be granted if all of the following conditions are satisfied:

1. The student is in good standing and needs only a final exam to complete the course. In particular, this means that no midterms are skipped, attendance to the discussion sessions was documented by quiz scores, and all projects received a satisfactory grade.

2. Some event (for which adequate documentation can be provided) prevented the student from doing a makeup final exam.

Note that these rules are from the university, and that the administration needs to approve incompletes.