Open book, open notes, open computer, but closed mouth! Write all answers on these sheets.

<table>
<thead>
<tr>
<th>question</th>
<th>1</th>
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<td>points</td>
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<td>maximum</td>
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<td>10</td>
<td>20</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>10</td>
<td>100</td>
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1. Give all Maple commands (NOT the output) to do the following.

(a) Make a list $L$ of 100 random digits. A digit belongs to $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(b) Select from $L$ all digits strictly larger than 5. Call this list $sL$.

(c) Compute $n$, the number of elements in $sL$.

(d) Use the digits in $sL$ to make a natural number with $n$ digits. Starting from the left (as usual), the $i$th digit in the number is the $i$th element of $sL$. 
2. For given \( k \) and \( n \), the function \( f \) computes \( kn^{n-k-1} \).

(a) Give Maple command to define the function \( f \).

(b) What is \( f(23,21) \)?

3. The number \( T(n,k) \) is defined along the following rules.

For \( k = 0 \) and \( n = 0 \): \( T(0,0) = 1 \). For \( k = 0 \) and \( n > 0 \): \( T(n,0) = 0 \).

For all \( n \geq k \geq 1 \):

\[
T(n,k) = \sum_{i=0}^{n-k} \binom{n-k}{i} T(n-1,k-1+i).
\]

(a) Write an efficient recursive Maple procedure to compute \( T(n,k) \):

(b) How many digits does \( T(44,1) \) have?
4. Explain the difference between \texttt{diff} and \texttt{implicitdiff}.
   Give a good example of both commands to illustrate their use.

5. A comet approaches earth in an ellipsoid path, defined by \( \frac{(x-2)^2}{6} + \frac{y^2}{2} - 1 = 0 \).
The equation \( x^2 + y^2 - 0.05 = 0 \) represents earth.
   (a) Make a plot of earth and the path of the comet, it must look like
   
   \begin{center}
   \includegraphics[width=0.8\textwidth]{plot}
   \end{center}

   Do \textbf{NOT} give the commands to make the plot, but write below
   
   i. the ranges for \( x \): \\
   ii. the ranges for \( y \): \\

   (b) We want to know the locations where the comet is closest to earth. Give the Maple commands to set up the polynomial system to find those locations.

   (c) Give the Maple command to bring this system into triangular form. Do \textbf{NOT} solve the system, but explain how many solutions the system has and interpret the solutions referring to the plot above.
6. Consider the curve defined by 
   \[(x^2 + y^2)^2 + 3x^2y - y^3 = 0.\]
   Give the Maple commands (not the output)
   (a) to plot this curve for \((x, y) \in [-2, +2] \times [-2, +2]:\)
   
   (b) to transform the equation into polar coordinates:

   (c) to plot this curve in polar coordinates:

7. Damped harmonic motion, expressing displacement \(y(t)\) as a function of time \(t\), with
   constants \(c\) and \(k\), leads to the equation
   \[
   \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = 0.
   \]
   Give all Maple commands (not the output)
   (a) to compute a tenth order series solution:

   (b) to convert this solution into a function. The parameters for this function are \(t\),
   the initial displacement \(y(0)\) and velocity \(y'(0)\). When evaluated, the function
   returns a polynomial in \(c\) and \(k\).