First Midterm Exam MCS 320 Jan Verschelde Wednesday 17 February 2016

The exam is open book, open notes and open computer.
All your answers to the questions below must be *hand written*, submitted on paper.

1. Consider $\sqrt{3}$. Compute the first ten terms in the continued fraction expansion.
   Write the last element in the corresponding list of convergents.

   Compare the floating-point approximation of this rational approximation for $\sqrt{3}$ with the floating-point approximation for $\sqrt{3}$. How many decimal places in the rational approximation are correct?

   Give the floating-point approximation of the rational approximation for $\sqrt{3}$.
   Write only those decimals that are correct.

2. Consider the statements `var('x')` and `QQ['x']`.

   What is the main difference between the roles of `'x'` after these statements?

   Start your answer with a precise description on the effect of each statement.
   Illustrate the difference. When should you use `var('x')`, when `QQ['x']`?
   For example, what can you do after `QQ['x']` but not after `var('x')`?

3. What is the fastest way to evaluate the expression `f = x*y^2 + 3*x^2*y - 2*x` with hardware arithmetic?

   Time the evaluation of the expression `f` at `(3.0, 4.0)`, and
   time the fastest way, also at `(3.0, 4.0)`. For both experiments, write the timings.

4. Consider the polynomial $p = x^5 - 2x^4 - 10x^3 + 20x^2 + 16x - 32$ over the field $\mathbb{Q}$ of rational numbers. Add sufficiently many formal roots to $\mathbb{Q}$ so you can write p as a product of linear polynomials.
   Give all relevant Sage commands and the final factorization for p.

5. Consider the expression $q = \dfrac{3x^2y^5}{x - 2y}$. Draw the expression tree for q.

6. Explain the difference between simultaneous and sequential substitution.
   Give an example of both forms of substitution.

7. Consider the polynomial p with integer coefficients in x and y,

$$p = x^3y^2 + 2x^2y^3 + 3x^2y + 4xy^3 + 5xy^2 + 6xy + 7y^3.$$

   Give the Sage commands to convert p (not retype) into the form

$$y^2x^3 + (2y^3 + 3y)x^2 + (4y^3 + 5y^2 + 6y)x + 7y^3.$$