Review of the first two parts of the course

In the first two parts of the course we learned about advanced number systems and about expression manipulation.

As the final exam happens in a computer lab, all worksheets and lecture notes will still be available to you via the course web site. However, as there is always a slim chance that the network or web server fails, it is strongly recommended to have a well organized folder with lecture notes.

Below is a first, preliminary list of fresh questions to review. Consider also the review questions for the midterms, the quizzes, midterms, and homework assignments.

Review of Part One: First Steps with Sage

1. Explain the difference between RR and RDF.

   Illustrate with an example.

2. Give a list of ten rational approximations for $\sqrt{3}$, accurate with 2, 3, up to 11 decimal places.

3. Consider $\mathbb{Z}_{31} = \{ 0, 1, 2, \ldots, 30 \}$ and answer the following questions:

   (a) What is the multiplicative inverse of 7 in $\mathbb{Z}_{31}$?

   (b) Show that $p = 15x^5 + 4x^4 + 23x^3 + 26x^2 + 6x + 1$ is irreducible over $\mathbb{Z}_{31}$.

4. Give all Sage commands to write $e^{I\frac{2\pi}{k}}$ as $\cos(2\frac{\pi}{k}) + I\sin(2\frac{\pi}{k})$.

5. Consider the expression $q = \cos(x^3 - 1) + 3\sin(y) - z^7$. Draw the expression tree for q and give all Sage commands you used to make the drawing.

6. Show how you can save a matrix of floating-point numbers from a Sage session to file, clear the variable that refers to the matrix, and load the matrix from file into Sage.

7. Generate optimized code to evaluate $p = 79x^{298}+56x^{205}+49x^{164}+63x^{121}+57x^{119}-59x^{42}$. How many aritmetical operations are needed to evaluate p? Compare with the cost of a direct evaluation of p.

Review of Part Two: Polynomials and Rational Expressions

8. Draw the internal representation of $p = xy(x - y)$.
   Give also the Sage commands used to obtain your drawing.

9. Consider the polynomial $p = x^3 - x - 2$ and give all Sage commands following questions:

   (a) to write p as an **exact** product of linear factors, with exact complex numbers;

   (b) to compute a **numerical** factorization of p over the complex numbers;

   (c) to define a **symbolic** (i.e.: formal) factorization of p,
   declaring sufficiently many roots.

10. Give all Sage commands to transform $(x - y)(x + y)$ into $(x + y)x - (x + y)y$.

11. Consider the rational expression $r = \dfrac{79x^5 + 56x^4 + 49x^3 + 63x^2 + 57x - 59}{45x^5 - 8x^4 - 93x^2 + 43x - 62}$.

    Convert r into a form which is more efficient to evaluate. Compare the number of arithmetical operations needed to evaluate r in this more efficient form with the number of arithmetical operations needed to evaluate r in its given form.

12. Explain why normal forms are so important to symbolic computation.
    What can we do if a normal form is too expensive to compute?
    Illustrate with a good example.

FINAL EXAM is on Thursday 5 May 2016, 10:30-12:30PM

In case of a scheduling conflict with another final exam, please let me know as soon as possible so we can schedule a makeup.