Review of parts three and four of the course

In part three of the course we saw how to define functions and explored Sage's capabilities in differentiation and integration. We discussed Sage's data structures and solved equations in the fourth and final part on Sage. As the final exam happens in a computer lab, all worksheets and lecture notes will still be available to you via the course web site. However, as there is always a slim chance that the network or web server fails, it is strongly recommended to have a well organized folder with lecture notes. Below is a first, preliminary list of fresh questions to review. Consider also the review questions for the midterms, the quizzes, midterms, and homework assignments.

13. The arc length of continuous function $f(x)$ over an interval $[a, b]$ can be defined as $\int_a^t \sqrt{1 + [f'(x)]^2}$.

    (a) Compute the arc length of the positive half of the unit circle, i.e.: $f(x) = \sqrt{1 - x^2}$ (answer $= \pi$).

    (b) Create a function (call it `arc_length`) in t which returns a 10-digit floating-point approximation of the arc length of the positive half of the circle, for $x \in [0, t]$.

14. Consider the recurrence relation

    $$h(n) = 5h(n-1) - 6h(n-2), \quad \text{for } n \geq 2, \quad \text{with } h(0) = 1 \text{ and } h(1) = -2.$$

    (a) The generating function $g(x) = \frac{1-7x}{1-5x+6x^2}$ defines $h(n)$ as the coefficient with x^n in the Taylor expansion of $g(x)$. Use $g(x)$ to define h as a function (call it `t`) of n which gives the value of $h(n)$.

    (b) Write a function to compute $h(n)$, directly using the recurrence relation from above. Make sure your function can compute $h(120)$. Compare with the result of (a).

15. The Legendre polynomials are defined by

    $$P_0(x) = 1, \quad P_1(x) = x, \quad P_n(x) = \frac{2n-1}{n} x P_{n-1}(x) - \frac{n-1}{n} P_{n-2}(x), \text{ for } n \geq 2.$$

    Write a efficient recursive function `legendre` to compute $P_n(x)$. The function `legendre` takes on input the degree n and the variable x.
    Compare the output of your `legendre`(50,x) with the `legendre_T`(50,x).

16. Suppose we want to plot the curve $x^4 + x^2y^2 - y^2 = 0$ for x and y both between -1 and $+1$.
    (a) Sampling this curve as given in rectangular coordinates, how many samples do we need to take from the curve to obtain a nice plot?
    (b) Convert the curve into polar coordinates and plot. Give all commands used to obtain the plot. How many samples of the curve are needed here?
17. Solve $x^2a^2 - 2x^2a - 3x^2 - xa^2 + 4xa - 3x + a^2 + 2a - 15$ for x for all values of the parameter a. Be as complete as possible in your description of the solution.
18. Find the point with real coordinates on the curve $xy - 2x + 3$ closest to the origin.
19. How many real solutions does the system $\left\{ \begin{array}{c} x^2 - 2y^2 - 1 = 0 \\ xy - 2x - 3 = 0 \end{array} \right.$ have?
20. Consider $y'' + 6y' + 13y = 0$, with $y(\pi/2) = -2$ and $y'(\pi/2) = 8$.
    (a) Find an exact solution to this initial value problem and use this to create a function `s` which returns a numerical 10-digit floating-point approximation of the solution.
    (b) Solve this initial value problem numerically. Compare the solution with the value for $y(2)$ and also with `s(2)` obtained in (a).

FINAL EXAM is on Thursday 5 May 2016, 10:30-12:30PM

In case of a scheduling conflict with another final exam, please let me know as soon as possible so we can schedule a makeup.