

# Defining Mathematical Functions

## 1 Expressions and Functions

- functions are expressions
- operations on mathematical functions

## 2 Step Functions

- discontinuous functions
- deltas and staircases

MCS 320 Lecture 19  
Introduction to Symbolic Computation  
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# symbolic evaluation of expressions

Expressions are callable objects. Evaluation is numeric or symbolic.

- numerical evaluation replaces the input by numbers,
- symbolic evaluation *substitutes* the input by *symbols*.

We can go back and forth between functions and expressions:

- 1 The substitution of an expression  
turns an expression into a function.
- 2 The symbolic evaluation of a function  
turns a function into an expression.

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# derivatives and antiderivatives

We view functions as *mathematical* functions.

- We can compute derivatives with respect to the input variables of a function.

For a function  $f$ , which depends on  $x$ , the *derivative*

$$\frac{d}{dx}f = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

is the instantaneous rate of change of  $f$  at  $x$ .

- The *antiderivative* is the opposite of the derivative:

$$\int \frac{d}{dx}f = f + C,$$

for some constant  $C$  which does not depend on  $x$ .

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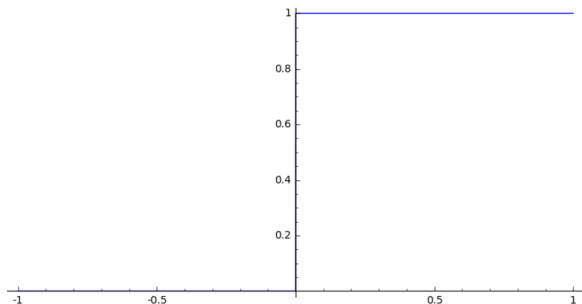
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- **discontinuous functions**
- deltas and staircases

# the unit step function



$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Common definitions for  $f(0)$  are 0, 1, or  $1/2$ .

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## deltas and staircases

- The derivative of the unit step is the Dirac delta function.  
The Kronecker delta is another well known discontinuous function.
- Suppose we want to define a 3-step staircase function  $f(x)$ , defined by the following four rules:
  - 1  $f(x) = 0$ , if  $x < 0$ .
  - 2  $f(x) = 1/3$ , if  $x \in [0, 1)$ .
  - 3  $f(x) = 2/3$ , if  $x \in [1, 2)$ .
  - 4  $f(x) = 1$ , if  $x \geq 2$ .
- The staircase is an example of a *piecewise function*.