Better Sorting Algorithms

1. Set Up
   - sorting a vector of pairs

2. Shell Sort
   - the Shell sort algorithm
   - C++ code for Shell sort

3. Merge Sort
   - the merge sort algorithm
   - C++ code for merge sort

4. Heap Sort
   - the heap sort algorithm
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MCS 360 Lecture 31
Introduction to Data Structures
Jan Verschelde, 8 April 2020
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Consider `vector<pair<int,char>> v` as a frequency table. We sort only on the `int` key.

5 random items: (82,i)(42,d)(42,x)(54,r)(31,z)

With `char` as second data field we can check if the sort is stable. In a stable sort, equal keys retain their relative order.

We assume we sort container with *random-access iterator*. For convenience: use subscripting operator `[]` on vectors.

We generate random vectors of 2-digit keys.
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Shell Sort

The idea of Donald Shell is to apply insertion sort to smaller subsequences separated by a gap value. For example, when gap equals 4:

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\end{array}
\rightarrow
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\end{array}
\]

First, sort four sequences: \([0, 4, 8], [1, 5, 9], [2, 6], \text{ and } [3, 7]\). Then, repeat with smaller value of gap.

Start with gap value equal to \(n/2\). Divide each time gap by 2.2, at end set gap to 1.

The division factor of 2.2 appears to be empirically to give performance of \(O(n^{5/4})\).

If the value for gap is \(2^k - 1\), then the cost is \(O(n^{3/2})\).
running an example

5 random items: (68,q)(79,f)(65,u)(58,j)(76,b)

insertion sort starts at 0 with gap 2
after 2 <-> 0: (65,u)(79,f)(68,q)(58,j)(76,b)

insertion sort starts at 1 with gap 2
after 3 <-> 1: (65,u)(58,j)(68,q)(79,f)(76,b)

insertion sort starts at 0 with gap 1
after 1 <-> 0: (58,j)(65,u)(68,q)(79,f)(76,b)

after 4 <-> 3: (58,j)(65,u)(68,q)(76,b)(79,f)

the sorted vector: (58,j)(65,u)(68,q)(76,b)(79,f)
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void insertion_sort
( vector< pair<int,char> >& v,
  int start, int gap )
{
    for(int i=start+gap; i<v.size(); i=i+gap)
    {
      int j=i-gap;
      while((j >= start)
 années ((v[i].first < v[j].first)) j=j-gap;
      j = j + gap;
      if(j < i)
      {
        pair<int,char> tmp = v[i];
        for(int k=i-gap; k>=j; k=k-gap)
          v[k+gap] = v[k];
        v[j] = tmp;
      }
    }
}
void sort ( vector< pair<int,char> >& v )
{
    int gap = v.size()/2;

    while(gap > 0)
    {
        for(int i=0; i<gap; i++)
            insertion_sort(v,i,gap);

        if(gap == 2)
            gap = 1;
        else
            gap = int(gap/2.2);
    }
}
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Merge Sort

To sort a sequence of $n$ elements, if $n > 1$:

1. split sequence in two equal halves,
2. sort first and second half,
3. merge the sorted halves.

Merge sort works on tapes, external data files.

Some data sets are too large to be stored entirely in internal memory.
running an example

4 random items : (15, m) (90, r) (63, p) (94, w)
in : (15, m) (90, r) (63, p) (94, w)
in : (15, m) (90, r)
in : (15, m)
out : (15, m)
in : (90, r)
out : (90, r)
out : (15, m) (90, r)
in : (63, p) (94, w)
in : (63, p)
out : (63, p)
in : (94, w)
out : (94, w)
out : (63, p) (94, w)
out : (15, m) (63, p) (90, r) (94, w)
the sorted vector : (15, m) (63, p) (90, r) (94, w)
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```cpp
vector< pair<int,char> > merge(
  vector< pair<int,char> > u,
  vector< pair<int,char> > v)
{
  vector< pair<int,char> > w;
  int i = 0;
  int j = 0;
  while((i < u.size()) && (j < v.size()))
    if(u[i].first <= v[j].first)
      w.push_back(u[i++]);
    else
      w.push_back(v[j++]);
  while(i < u.size()) w.push_back(u[i++]);
  while(j < v.size()) w.push_back(v[j++]);

  return w;
}
```
function merge_sort

vector< pair<int,char> > merge_sort
( vector< pair<int,char> >& v )
{
    vector< pair<int,char> > w;
    if(v.size() < 2)
        w = v;
    else
    {
        int middle = v.size()/2;
        vector< pair<int,char> > a,b,s_a,s_b;
        for(int i=0; i<middle; i++) a.push_back(v[i]);
        for(int i=middle; i<v.size(); i++) b.push_back(v[i]);
        s_a = merge_sort(a);
        s_b = merge_sort(b);
        w = merge(s_a,s_b);
    }
    return w;
}
Cost Analysis

The cost of a sort depends on
1. the number of comparisons,
2. the number of assignments.

How many levels in the recursion?
For a sequence of size $n$: split $\log_2(n)$ times.

Splitting and merge takes $O(n)$ operations:

$$\frac{n}{2} + \frac{n}{4} + \cdots + 2 + 1 = n - 1.$$ 

Best case? Worst case?
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a heap as a vector

A heap is a binary tree:

For node at $p$: left child is at $2p + 1$, right child is at $2p + 2$. Parent of node at $p$ is at $(p - 1)/2$. 
storing integer numbers

pushing 21 60 17 65 90 27 70
sorting with a heap
popping largest element

```
90
  /
65  70
  /  /  \\
21 60 17 27

70
  /
65  27
  /  /  \\
21 60 17

65
  /
60  27
  /  /  \\
21 27 17

27
  /
17
```

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running an example

5 random items: (81, x)(74, m)(78, i)(78, j)(50, i)

insert i=0: (81, x)

insert i=1: (81, x)(74, m)

insert i=2: (81, x)(74, m)(78, i)

insert i=3: (81, x)(78, j)(78, i)(74, m)

insert i=4: (81, x)(78, j)(78, i)(74, m)(50, i)

remove i=0: (78, i)(78, j)(50, i)(74, m)

remove i=1: (78, j)(74, m)(50, i)

remove i=2: (74, m)(50, i)

remove i=3: (50, i)

remove i=4:

the sorted vector: (50, i)(74, m)(78, j)(78, i)(81, x)
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vector< pair<int,char> > sort
    ( vector< pair<int,char> >& v )
{
    vector< pair<int,char> > h,w;
    int bottom = -1;

    for(int i=0; i<v.size(); i++)
        insert(h,bottom,v[i]);

    for(int i=0; i<v.size(); i++)
    {
        w.insert(w.begin(),h[0]);
        remove(h,bottom);
    }
    return w;
}
Cost Analysis

Inserting and removing an element:
- depends on the depth of the binary tree,
- the heap is a complete binary tree,
- the depth is $\log_2(n)$.

Therefore: the cost for insert/remove is $O(\log_2(n))$.

We have $n$ elements, so the cost is $2n\log_2(n)$.

Best case? Worst case?
Summary + Exercises

More of chapter 10 on sorting algorithms with Shell sort, heap sort, and merge sort.

Exercises:

1. Modify the code for Shell sort to count the #comparisons and #swaps. Run at least 10 sorts of random sequences of sizes 100, 200, and 400. Do data fitting on the counts. Do you observe $O(n^{3/2})$?

2. Describe an inplace merge sort. Extra input parameters are the begin and end index in the vector.

3. Give an example (with 7 numbers) of the best and worst case for heap sort.