Expression Trees and the Heap

1. Binary Expression Trees
   - evaluating expressions
   - splitting strings in operators and operands
2. C++ Binary Tree of Strings
   - header files
   - defining the methods
3. the Heap or Priority Queue
   - a heap of integer numbers
   - the heap ADT and algorithms to push and pop
   - our class Heap with STL vector

MCS 360 Lecture 25
Introduction to Data Structures
Jan Verschelde, 11 March 2020

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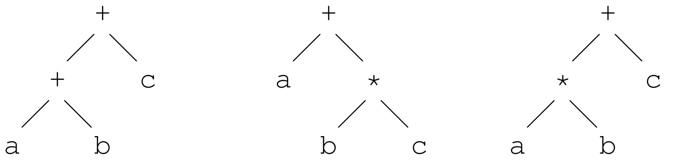
Binary Expression Trees

Expression trees store the evaluation order:

`a+b+c` `a+b*c` `a*b+c`



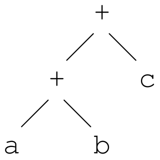
= `(a+b)+c` = `a+(b*c)` = `(a*b)+c`

As the children store the operands,
we first evaluate the expressions at the children.

running the program I

```
$ /tmp/strtree
Give expression : a+b+c
your expression : a+b+c
the expression tree :
+
  +
    a
    b
  c
postfix : ab+c+
$
```

a+b+c

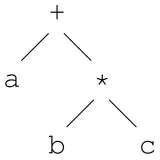


= (a+b)+c

running the program II

```
$ /tmp/strtree
Give expression : a+b*c
your expression : a+b*c
the expression tree :
+
  a
  *
    b
    c
postfix : abc*+
$
```

a+b*c

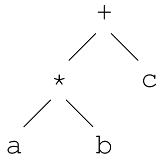


= a+(b*c)

running the program III

```
$ /tmp/strtree
Give expression : a*b+c
your expression : a*b+c
the expression tree :
+
  *
    a
    b
  c
postfix : ab*c+
$
```

a*b+c



= (a*b)+c

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expressions as strings

We consider expressions with symbolic operands.

`"a+b+c"` is split into `"a"`,`"+"`,`"b"`,`"+"`,`"c"`

We split a string into a vector of strings.

Recall:

- `find()` method on string,
- `push_back()` method on vector.

splitting strings

```
vector<string> split ( string e, string op )
{
   vector<string> r;
   int i = 0;
   if(e.find(op,i) == string::npos)
      r.push_back(e);
   else
   {
      int k;
      do
      {  k = e.find(op,i);
         r.push_back(e.substr(i,k-i));
         r.push_back(e.substr(k,1)); i = k+1;
      }
      while(!(e.find(op,i) == string::npos));
      r.push_back(e.substr(k+1));
   }
   return r;
}
```

splitting twice

We can split first on `"+"`, then on `"*"`.

```
$ /tmp/split
give expression : a+b*c
your expression : a+b*c
after split :
a
+
b*c
after second split :
 a + ( b * c )
$
```

Instead of using a vector of strings,
the split is executed recursively with a tree.

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a node

```
#ifndef __TREE_NODE_H__
#define __TREE_NODE_H__
#include <string>

struct Node
{
   std::string data; // operator or operand
   Node *left;       // pointer to left branch
   Node *right;      // pointer to right branch

   Node(const std::string& s,
        Node* left_ptr = NULL,
        Node* right_ptr = NULL) :
     data(s), left(left_ptr), right(right_ptr) {}

   virtual ~Node() {}
};
#endif
```

mcs360_binary_expression_tree.h

```
#ifndef __MCS360_BINARY_EXPRESSION_TREE_H__
#define __MCS360_BINARY_EXPRESSION_TREE_H__

#include "mcs360_binary_expression_node.h"

namespace mcs360_binary_expression_tree
{
   class Tree
   {
      private:

         Node *root;  // data member

         // construct tree from a node
         Tree(Node *r) : root(r) {}
```

public constructor methods

```
public:

   Tree() : root(NULL) {}

   Tree(const std::string& s,
        const Tree& left = Tree(),
        const Tree& right = Tree() ) :
     root(new Node(s,left.root,right.root)) {}
```

Note: a tree is a pointer to a node,
though a client of `Tree` does not see the `Node` type.

other public methods

```
Tree get_left() const;  // returns left child
// precondition: not is_left_null()
Tree get_right() const; // returns right child
// precondition: not is_right_null();

bool is_left_null() const;
// true if left child is null
bool is_right_null() const;
// true if right child is null

std::string get_data() const;
// returns data at node

void insert(std::string e);
void insert(std::string e, std::string op);
```

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selectors

```
#include "mcs360_binary_expression_tree.h"

namespace mcs360_binary_expression_tree
{
   Tree Tree::get_left() const {
      return Tree(root->left);
   }
   Tree Tree::get_right() const {
      return Tree(root->right);
   }
   bool Tree::is_left_null() const {
      return (root->left == NULL);
   }
   bool Tree::is_right_null() const {
      return (root->right == NULL);
   }
   std::string Tree::get_data() const {
      return root->data;
   }
```

the method insert()

```
void Tree::insert(std::string e)
{
   this->insert(e,"+");
}
void Tree::insert(std::string e, std::string op)
{
   using std::string;

   if(e.rfind(op) == string::npos)
   {
      if(op == "*")
         root = new Node(e);
      else
      {
         Tree S;
         S.insert(e,"*");
         root = S.root;
      }
   }
```

`insert()` continued

```
    else
    {
       int k = e.rfind(op);
       Tree L;
       L.insert(e.substr(0,k));
       Tree R;
       R.insert(e.substr(k+1));
       root = new Node(e.substr(k,1),L.root,R.root);
    }
}
```

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the Heap

A *complete* binary tree is a *heap* if

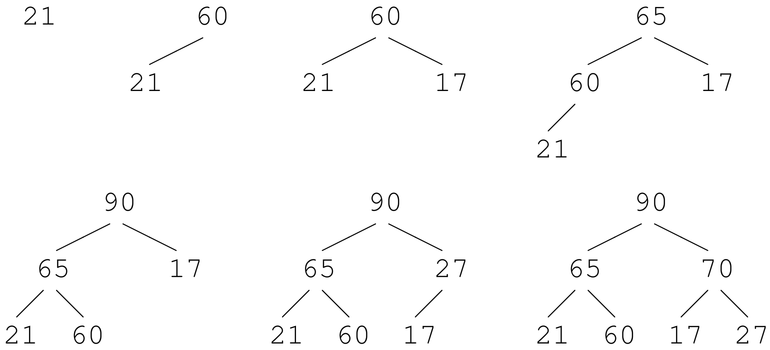
1. the root is the largest element; and
2. the subtrees are also heaps.

If the root is largest, we have a *max* heap.
If the root is smallest, we have a *min* heap.

The root is called the *top* of the heap.
The *bottom* of the heap is the rightmost element at the deepest level of the tree.

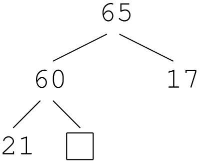
storing integer numbers

pushing `21 60 17 65 90 27 70`

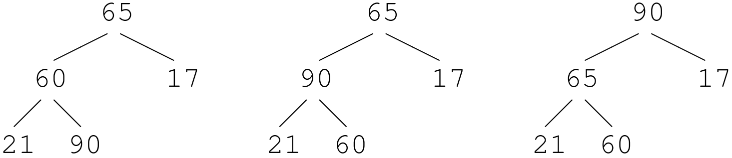


swapping numbers

Insert `90` into:



The bottom is `21`, the box marks the new bottom.
As long as child is larger than parent, we swap:



Important: #swaps is bounded by depth of the tree.
If n numbers on heap, then push() is $O(\log_2(n))$.

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the heap ADT

```
abstract <typename T> heap;
/* A heap is a complete binary tree where the data
   at a node is larger than any element in the subtrees. */

abstract bool empty ( heap h );
postcondition: true if the heap is empty,
  false if the heap is not empty.

abstract T top ( heap h );
precondition: not empty(h);
postcondition: top(h) is the largest element in the heap;

abstract T bottom ( heap h );
precondition: not empty(h);
postcondition: bottom(h) is the bottom element of the heap;
```

the heap ADT continued

We push to the bottom and pop from the top.

```
abstract T push ( heap h, T item );
postcondition: push(h) inserts the item
  in the heap h, maintaining the property
  of the heap with the item added.

abstract T pop ( heap h );
precondition: not empty(h);
postcondition: removes top(h) from the
  heap h, maintaining the property of
  the heap with the item removed.
```

pushing and popping an item into a heap

The algorithm to push an item into a heap:

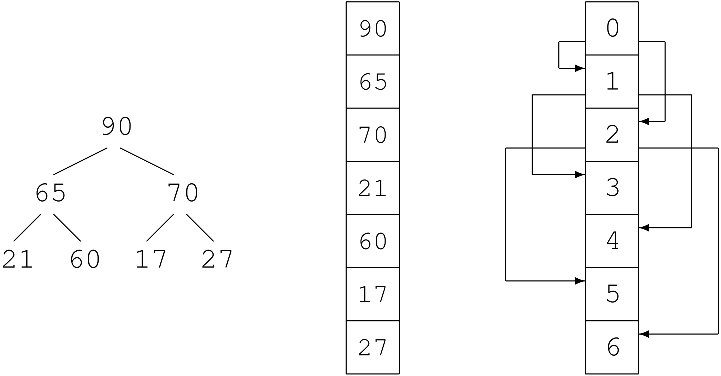
```
place the item at the new bottom
while the item is larger than the parent do
      swap the item with the parent.
```

The algorithm to pop an item from a heap:

```
remove the item, replace it with the bottom B
while B has larger children do
      swap B with its larger child.
```

The cost of the algorithm in linear in the depth of the tree, or equivalently, logarithmic in the number of items stored.

storing heap as vector



For node at p: left child is at $2p + 1$, right child is at $2p + 2$.
Parent of node at p is at $(p - 1)/2$.

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a class Heap in the file mcs360_integer_heap.h

```
#ifndef __MCS360_INTEGER_HEAP_H__
#define __MCS360_INTEGER_HEAP_H__

#include<vector>
#include<string>

namespace mcs360_integer_heap
{
   class Heap
   {
      private:

         std::vector<int> h;

         int index_to_bottom;
```

public methods

```
public:

   Heap(); // creates empty heap

   int size() const;
   // returns the size of the heap

   int top() const;
   // returns the top of the heap

   int bottom() const;
   // returns the bottom of the heap

   void push ( int n );
   // pushes n to the heap
```

file mcs360_integer_heap.cpp

```
#include "mcs360_integer_heap.h"

namespace mcs360_integer_heap
{
   Heap::Heap() {
      index_to_bottom = -1;
   }
   int Heap::size() const {
      return index_to_bottom+1;
   }
   int Heap::top() const {
      return h[0];
   }
   int Heap::bottom() const {
      return h[index_to_bottom];
   }
```

the method push()

```
void Heap::push( int n )
{
   if(h.size() > this->size())
      h[++index_to_bottom] = n;
   else
   {
      h.push_back(n);
      index_to_bottom++;
   }
   swap_from_bottom(index_to_bottom);
}
```

swapping elements

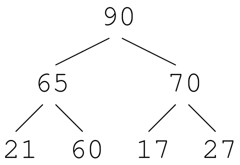
A private function member for `push()`:

```
void Heap::swap_from_bottom( int p )
{
   if(p == 0) return;

   int parent = (p-1)/2;
   if(h[parent] < h[p])
   {
      int t = h[p];
      h[p] = h[parent];
      h[parent] = t;
      swap_from_bottom(parent);
   }
}
```

converting to string

To write




```
the heap as vector: 90 65 70 21 60 17 27
the heap as tree:
90
  65
    21
    60
  70
    17
    27
```

writing to string

```
std::string Heap::to_tree_string( int k, int p )
{
   using std::ostringstream;
   ostringstream s;

   for(int i=0; i<k; i++) s << "  ";
   s << h[p] << std::endl;

   int left = 2*p+1;
   if(left > index_to_bottom) return s.str();
   s << to_tree_string(k+1,left);

   int right = 2*p+2;
   if(right > index_to_bottom) return s.str();
   s << to_tree_string(k+1,right);
   return s.str();
}
```

Summary + Exercises

Introduced expression trees and started §8.5 on the heap.

Exercises:

1. Describe the changes needed to the binary expression trees program to deal with subtraction and division. How would you handle brackets and nesting?
2. Implement the changes of the previous exercise.
3. Generate a random sequence of 10 numbers and draw the evolution of the tree (also for every swap) when pushing the numbers onto a heap.
4. Define an exception `EmptyHeap` to be thrown when `top()` or `bottom()` is applied to an empty heap.